1 Number Patterns

By studying this lesson you will be able to,

- identify the *n*th term of a given number pattern, and
- find any term of a number pattern when the *n*th term is given.

1.1 Number patterns and terms of a number pattern

Let us write the odd numbers from 3 to 11 in ascending order.

3, 5, 7, 9, 11

This is the number pattern of the odd numbers from 3 to 11 written in ascending order.

- When numbers are written in a certain order according to a specific method or rule, starting from a certain number, it is called a **number pattern**.
- Every number in a given number pattern is called a **term of the number pattern**.
- The first number of a number pattern is called the first term and the following numbers in order are called the second term, third term, fourth term, etc.
- Commas are used to separate the terms of a number pattern.

Let us consider again the number pattern 3, 5, 7, 9, 11 of the odd numbers from 3 to 11 written in ascending order.

The first term of the above pattern is 3 and the fourth term is 9. The last or the 5th term is 11. There are only five terms in this number pattern. Therefore the number of terms is finite.

Such number patterns, where the number of terms is finite, are called **finite number patterns**.

Let us write the even numbers starting from 2 in ascending order.

2, 4, 6, 8, …

You learnt in Grade 6 that this is the number pattern of the even numbers starting from 2 written in ascending order.

Since the exact number of terms in this number pattern cannot be specified, that is, since it is infinite, we cannot write down all its terms. Therefore, the first few terms are written such that the pattern can be identified, and as above, three dots are used to denote the rest of the terms.

Such number patterns where the number of terms is not finite, are called **infinite number patterns**.

Example 1

Write the terms of each of the following number patterns.

(i) The number pattern of the prime numbers between 1 and 17, written in ascending order.

(ii) The number pattern of the odd numbers starting from 1, written in ascending order.

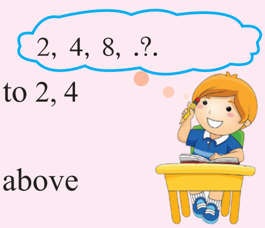
(iii) The number pattern starting from 1 and followed by the terms 2 and 1 written alternatively.

(i) 2, 3, 5, 7, 11, 13 (ii) 1, 3, 5, 7, 9, …

(iii) 1, 2, 1, 2, 1, 2, …

Note

Consider the number pattern 2, 4, 8,…



A number pattern with the first, second and third terms equal to 2, 4 and 8 respectively is given above.

We can easily write two different number patterns with the above first three terms.

(i) 2, 4, 8, 16, 32, 64, …
Here a term is multiplied by 2 to get the next term.

(ii) 2, 4, 8, 10, 20, 22, 44, …
Here, the second term is obtained by adding two to the first term, the third term is obtained by multiplying the second term by two, the fourth term is obtained by adding two to the third term, etc.

An important fact that can be learnt from this is that there can be more than one number pattern having the same first few terms.

Exercise 1.1

(1) Fill in the blanks.

(i) In the number pattern 1, 3, 5, 7, 9, …

the first term = ...
the second term = ...
the fourth term = ...

(ii) In the number pattern 4, 8, 12, 16, 20, …
the first term = ...
the second term = ...
the third term = ...

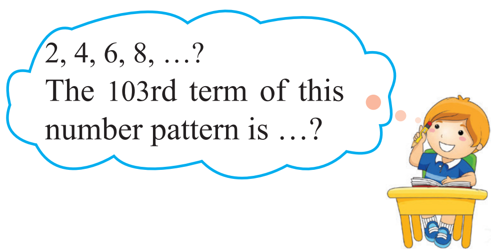
(2) Write the terms of each of the following number patterns.

(i) The number pattern of the even numbers between 1 and 9 written in ascending order.
(ii) The number pattern of the multiples of 6 from 6 to 36 written in ascending order.
(iii) The number pattern of the even numbers greater than 7 written in ascending order.
(iv) The number pattern of the prime numbers starting from 2 written in ascending order.

(3) Copy the below given statements in your exercise book and mark the correct statements with a ✓ and the incorrect statements with a ×.
(i) The terms of a number pattern have to be in ascending order.
(ii) The terms of a number pattern have to be different from each other.
(iii) If the 10th terms of two number patterns are different, then the two number patterns are different to each other.

1.2 The general term of a number pattern

Let us consider how we can easily find any term of a number pattern.



When the *n*th term of a number pattern is written as an algebraic expression in *n*, it is called the **general term of the number pattern**.

Using the general term, the value of any term in the number pattern can be found.

• The general term of the number pattern of the multiples of a number

➤ Let us consider the number pattern of the multiples of 2, starting from 2 and written in ascending order.

This number pattern is 2, 4, 6, 8, …

Although the terms from the fifth term onwards are not written, we know that the fifth term is 10, the sixth term is 12 and the seventh term is 14.

Let us find the nth term of this number pattern.
The table given below, shows how the value of each term is obtained.

Term	Value of the term	How the value of the term is obtained
First term	2	2×1
Second term	4	2×2
Third term	6	2×3
Fourth term	8	2×4
⋮	⋮	⋮
Tenth term	?	2×10
⋮	⋮	⋮
nth term	?	$2 \times n$
⋮	⋮	⋮

According to the 3rd column of the table, the nth term of the above number pattern is $2 \times n$; that is $2n$.

The nth term of this number pattern is $2n$. This is called the **general term** of this number pattern. By substituting suitable values for n in $2n$, we can obtain the values of the relevant terms.

The value of n in the general term of a number pattern should always be a positive integer.

The above number pattern is the same as **the even numbers starting from 2 and written in ascending order**.

- The general term of the number pattern of the even numbers starting from 2 and written in ascending order is $2n$.
- The general term of the number pattern of the multiples of 2 starting from 2 and written in ascending order is $2n$.

Example 1

In the number pattern of the multiples of 2 starting from 2 and written in ascending order,
(i) find the 11th term.
(ii) find the 103rd term.
(iii) find which term is 728.

(i) General term = $2n$

Since $n = 11$, the 11th term = $2 \times 11 = 22$

(ii) The 103rd term = 2×103

$= 206$

(iii) Since 728 is a multiple of 2, it should be a term of this number pattern. To identify which term it is, the general term should be equated to 728 and the value of n satisfying this equation should be found.

$$2n = 728$$
$$\frac{2n}{2} = \frac{728}{2}$$
$$n = 364$$

Accordingly, 728 is the 364th term of this number pattern.

➤ Let us consider the number pattern of the multiples of 3 starting from 3 and written in ascending order.

This number pattern is 3, 6, 9, 12, …

How the values of the terms of this number pattern are obtained is shown in the following table.

Term	Value of the term	How the value of the term is obtained
First term	3	3×1
Second term	6	3×2
Third term	9	3×3
Fourth term	12	3×4
⋮	⋮	⋮
Tenth term	?	3×10
⋮	⋮	⋮
nth term	?	$3 \times n$
⋮	⋮	⋮

According to the third column of this table, the nth term in this number pattern is $3 \times n$; that is $3n$.

The general term of the number pattern of the multiples of 3 starting from 3 and written in ascending order is $3n$.

Accordingly,

- the general term of the number pattern of the multiples of 4 starting from 4 and written in ascending order is $4n$.
- the general term of the number pattern of the multiples of 7 starting from 7 and written in ascending order is $7n$.

Example 2

The general term of the number pattern of the multiples of 3 starting from 3 and written in ascending order is $3n$.

(i) Find the 13th term of this number pattern.
(ii) Find which term 87 is of this number pattern.

(i) The general term of the number pattern of the multiples of 3 starting from 3 and written in increasing order is $3n$

The 13th term of this number pattern $= 3 \times 13 = 39$

(ii) $3n = 87$

Let us find the value of n that satisfies this equation.

$$\frac{3n}{3} = \frac{87}{3}$$
$$n = 29$$

$\therefore$ 87 is the 29th term of this number pattern.

Example 3

In the number pattern of the multiples of 4 starting from 4 and written in ascending order, with general term $4n$,

(i) what is the 10th term?
(ii) what is the 11th term?
(iii) which term is 100?
(iv) is 43 a term of this number pattern? What are the reasons for your answer?

(i) The general term of the number pattern of the multiples of $4 = 4n$

10th term $= 4 \times 10$
$= 40$

(ii) The general term of the number pattern of the multiples of $4 = 4n$

11th term $= 4 \times 11$
$= 44$

(iii) Since the general term of the number pattern of the multiples of 4 is $4n$,

$$4n = 100$$
$$\frac{4n}{4} = \frac{100}{4}$$
$$n = 25$$

$\therefore$ 100 is the 25th term.

(iv) When $4n = 43$

$$\frac{4n}{4} = \frac{43}{4}$$
$$n = 10\frac{3}{4} \text{ (This is not a positive integer)}$$

$\therefore$ 43 cannot be a term of this number pattern.

43 is not a multiple of 4. Therefore, it can be said that 43 is not a term of this number pattern.

Exercise 1.2

(1) Copy the table given below and complete it.

Number pattern	First term	General term
5, 10, 15, 20, ...		
10, 20, 30, 40, ...		
8, 16, 24, 32, ...		
7, 14, 21, 28, ...		
12, 24, 36, 48, ...		
1, 2, 3, 4, ...		

(2) Write the number pattern of the multiples of 5 between 3 and 33 written in ascending order.

(3) In the number pattern 11, 22, 33, 44, … of the multiples of 11 starting from 11 and written in ascending order,

(i) what is the general term?

(ii) what is the 9th term?

(iii) which term is 121?

(4) In the number pattern 9, 18, 27, 36, … of the multiples of 9 starting from 9 and written in ascending order,

(i) what is the general term ?

(ii) what is the 11th term?

(iii) which term is 270?

(5) In the number pattern with general term $100n$,

(i) what is the 11th term?

(ii) which term is 500?

(6) What is the smallest multiple of 3 larger than 100? Which term is it in the number pattern of the multiples of 3 starting from 3?

(7) What is the nth term (general term) of the pattern of the even numbers greater than 1 but less than 200 written in ascending order? The smallest value of n is 1. What is its largest value?

(8) It has been estimated that in a country having a population of 2 million people, the population will increase by 2 million people every 25 years. Estimate the population of the country in 200 years.

• The general term of the pattern of the odd numbers

You have learnt earlier that odd numbers are numbers which have a remainder of 1 when divided by 2.

1, 3, 5, 7, ... is the pattern of the odd numbers starting from 1 written in ascending order.

Since we obtain a remainder of 1 when an odd number is divided by 2, we should obtain an odd number when 1 is subtracted from any multiple of 2.

Accordingly, let us identify how the pattern of the odd numbers is developed by considering the following table.

Term	Multiples of 2	Multiples of 2 - 1	Odd number
First term	$2 = 2 \times 1$	$(2 \times 1) - 1$	$2 - 1 = 1$
Second term	$4 = 2 \times 2$	$(2 \times 2) - 1$	$4 - 1 = 3$
Third term	$6 = 2 \times 3$	$(2 \times 3) - 1$	$6 - 1 = 5$
⋮	⋮	⋮	⋮
Tenth term	$20 = 2 \times 10$	$(2 \times 10) - 1$	$20 - 1 = 19$
⋮	⋮	⋮	⋮
nth term	$2n = 2 \times n$	$(2 \times n) - 1$	$2n - 1$
⋮	⋮	⋮	⋮

The general term of the pattern of the odd numbers starting from 1 and written in ascending order can be expressed in terms of the general term of the pattern of even numbers starting from 2, written in ascending order.

The general term of the pattern of the odd numbers starting from 1 and written in ascending order is $2n - 1$.

Example 4

In the pattern of the odd numbers 1, 3, 5, 7, … starting from 1,

(i) what is the general term?
(ii) what is the 72nd term?
(iii) which term is 51?

(i) Since this is the pattern of the odd numbers starting from 1, the general term is $2n - 1$.

(ii) When $n = 72$, the seventy second term $= 2 \times 72 - 1$
$= 144 - 1$
$= 143$

(iii) Let us take that 51 is the nth term of this number pattern.

Then, $2n - 1 = 51$

$$2n - 1 + 1 = 51 + 1$$
$$2n = 52$$
$$\frac{2n}{2} = \frac{52}{2}$$
$$n = 26$$

51 is the 26th term of this number pattern.

Exercise 1.3

(1) In the pattern of the odd numbers starting from 1 and written in ascending order,

(i) what is the 12th term?
(ii) what is the 15th term?
(iii) which term is 89?
(iv) which term is the greatest odd number less than 100?

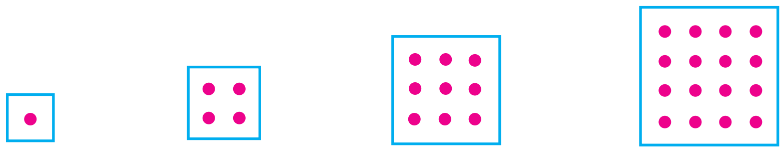
(2) Find the value of the sum of the 34th term of the pattern of the even numbers starting from 2 and the 34th term of the pattern of the odd numbers starting from 1.

General term of the pattern of the square numbers

You have learnt in Grade 6 that 1, 4, 9, 16, … are the square numbers written in ascending order.

This pattern represented by square shaped figures consisting arrangements of dots is given below.

First term   Second term   Third term   Fourth term



1×1 2×2 3×3 4×4

1^2 2^2 3^2 4^2

The pattern of the square numbers is developed as follows.

First term $= 1 \times 1 = 1^2 = 1$

Second term $= 2 \times 2 = 2^2 = 4$

Third term $= 3 \times 3 = 3^2 = 9$

$\vdots$

Tenth term $= 10 \times 10 = 10^2 = 100$

$\vdots$

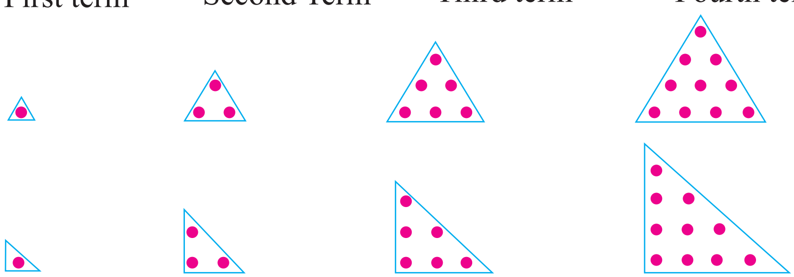
nth term $= n \times n = n^2$

$\therefore$ The general term of the pattern of the square numbers starting from 1 and written in ascending order is n^2.

The general term of the pattern of the triangular numbers

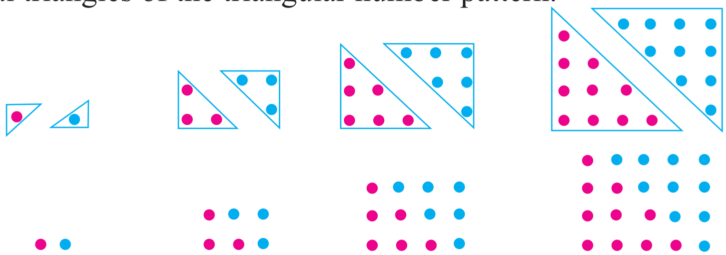
You have learnt in Grade 6 that 1, 3, 6, 10, 15, … are the triangular numbers written in ascending order. They can be represented by dots in both the ways given below.

First term   Second Term   Third term   Fourth term



1 $1 + 2 = 3$ $1 + 2 + 3 = 6$ $1 + 2 + 3 + 4 = 10$

Rectangular shaped arrangements of dots that have twice the number of dots as each corresponding figure in the triangular number pattern can be obtained by joining together two equal triangles of the triangular number pattern.



Number of rows	1	2	3	4
Number of columns	2	3	4	5
Total number of dots	1×2	2×3	3×4	4×5
Triangular number	$\frac{1 \times 2}{2} = 1$	$\frac{2 \times 3}{2} = 3$	$\frac{3 \times 4}{2} = 6$	$\frac{4 \times 5}{2} = 10$

Therefore, the pattern of triangular numbers is as follows.

First term $= \frac{1 \times 2}{2} = 1$

Second term $= \frac{2 \times 3}{2} = 3$

Third term $= \frac{3 \times 4}{2} = 6$

Fourth term $= \frac{4 \times 5}{2} = 10$

$\vdots \qquad \vdots \qquad \vdots$

Tenth term $= \frac{10 \times 11}{2} = 55$

$\vdots \qquad \vdots \qquad \vdots$

nth term $= \frac{n \times (n + 1)}{2} = \frac{n\,(n + 1)}{2}$

The general term of the triangular number pattern starting from 1 and written in ascending order is $\frac{n\,(n + 1)}{2}$.

Exercise 1.4

(1) What is the 10th term of the square number pattern starting from 1 and written in ascending order?

(2) What is the 10th term of the triangular number pattern starting from 1 and written in ascending order?

(3) A certain number greater than 1 and less than 50, which is a term of the square number pattern starting from 1 and written in ascending order, is also a term of the triangular number pattern starting from 1 and written in ascending order.

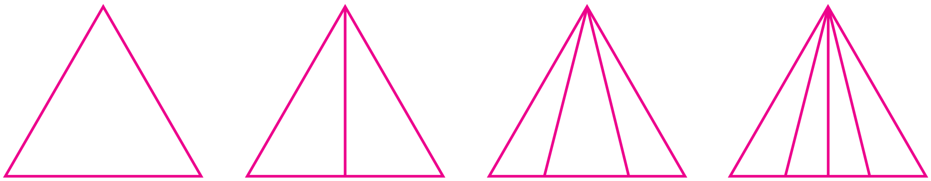
(i) What is this term?

(ii) Which square number is it?

(iii) Which triangular number is it?

(4) "The sum of the 14th and 15th terms of the triangular number pattern starting from 1 is a square number". Show that this statement is true and find which term it is of the square number pattern.

(5) Write the total number of triangles in each figure in order and see whether you can identify the pattern.



(i) (ii) (iii) (iv)

The pattern of the total number of triangles in the figures in the given order, is identical to the pattern of triangular numbers starting from 1 and written in ascending order. Find the total number of triangles in the 8th figure that is drawn according to this pattern.

(6) Sayuni buys a till and starts saving money by putting one rupee into it on the first day. On the second day she puts 2 rupees, on the third day 3 rupees and so forth. How much money is in the till by the end of the 10th day?

Miscellaneous Exercise

(1) In the pattern of the odd numbers starting from 1, commencing from the first term, if the first two terms, then the first three terms, then the first four terms are added and continued accordingly, a special type of numbers is obtained.

(i) What is the special name given to these numbers?

(ii) Find the number that is obtained if 15 of these terms are added in order starting from the first term.

(2) Milk tins brought to a shop to be sold were arranged on a rack in the following manner.

- 10 tins on the lowest shelf and every other shelf having one tin less than the number on the shelf below it.1 tin on the topmost shelf.

(i) Find the number of milk tins that were brought to the shop.

(ii) All the milk tins on the four topmost shelves were sold within two weeks. Find the number of milk tins that were sold.

(3) What is the sum of the integers from 1 to 30?

What is the difference between a set of numbers and a number pattern?

The pattern of the even numbers between 1 and 9 written in ascending order is 2, 4, 6, 8.

If these four numbers are written in descending order as 8, 6, 4, 2, we obtain a different number pattern.
A is the set of even numbers between 1 and 9.

We can write the set of even numbers between 1 and 9 as follows.
A = {2, 4, 6, 8} = {6, 4, 8, 2} = {8, 6, 4, 2}

Whatever order the numbers 2, 4, 6, 8 are written within brackets, we obtain the same set. Elements of a set are not named as the first element, the second element, etc.

$\therefore$ Although {2, 4, 6, 8} and {8, 6, 4, 2} are the same set, the number pattern 2, 4, 6, 8 is not equal to the number pattern 8, 6, 4, 2.

Summary

- The expression in n obtained for the nth term of a number pattern is called its general term.
- The value of n in the general term of a number pattern should always be a positive integer.
- The general term of the number pattern of the even numbers starting from 2 and written in ascending order is $2n$.
- The general term of the number pattern of the odd numbers starting from 1 and written in ascending order is $2n - 1$.
- The general term of the pattern of the square numbers starting from 1 and written in ascending order is n^2.
- The general term of the triangular number pattern starting from 1 and written in ascending order is $\frac{n(n+1)}{2}$.

Think

(1) Can you construct three different number patterns with 1, 2, 4 as the first three terms?
If you can, then write the next two terms of each of those number patterns.

2 Perimeter

By studying this lesson you will be able to,

- calculate the perimeters of composite rectilinear plane figures composed of two similar or different types of plane figures from equilateral triangles, isosceles triangles, squares and rectangles, and
- solve problems involving the perimeters of composite rectilinear plane figures.

2.1 Perimeter

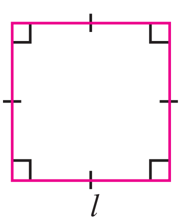
Suppose we need to find the length around a rectangular plot of land. For this we need to obtain the sum of the lengths of all four sides of the plot.

The measurement that is thus obtained is said to be the perimeter of the plot of land.

You have learnt earlier that, the sum of the lengths of all the sides of a closed rectilinear plane figure is called its **perimeter**.

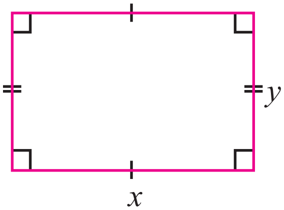
Now let us recall some of the formulae you learnt in Grades 6 and 7 that can be used to find the perimeter of certain plane figures.

- If the perimeter of a square of side length l units is p units, then



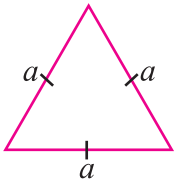
$$p = l + l + l + l$$
$$p = 4l$$

- If the perimeter of a rectangle of length x units and breadth y units is p units, then



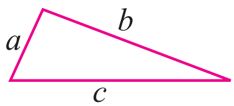
$$p = x + y + x + y$$
$$p = 2x + 2y$$

- If the perimeter of an equilateral triangle of side length a units is p units, then



$$p = a + a + a$$
$$p = 3a$$

- If the perimeter of a triangle with side lengths a, b and c units is p units, then



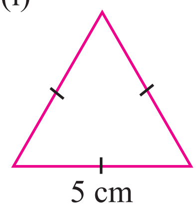
$$p = a + b + c$$

Do the following review exercise to revise what you have learnt.

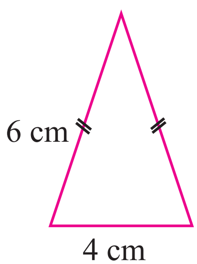
Review Exercise

(1) Find the perimeter of each of the figures given below.

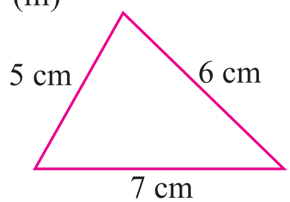
(i)



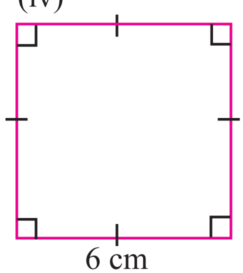
(ii)



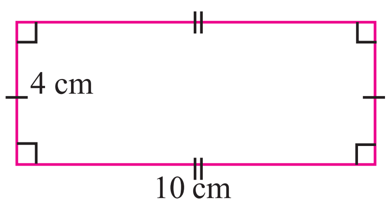
(iii)



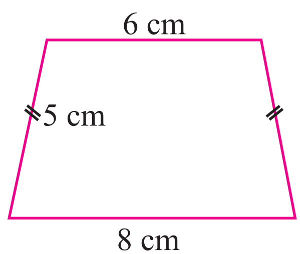
(iv)



(v)



(vi)



(2) The perimeter of a square shaped wall tile is 160 cm. How many such tiles are needed for one lengthwise row of a wall of length 4 m, if the tiles are to be fixed without any gaps between them?

(3) If the perimeter of a rectangular shaped paddy field of length 40 m is 130 m, find its breadth.

(4) The length of a rectangular shaped wall tile is greater than its breadth by 10 cm. If the breadth of the tile is 15 cm, find its perimeter.

(5) There are two pieces of wire of length 60 cm each. Amali makes an equilateral triangle by bending one of these pieces of wire. Sandamini makes a square with the other piece of wire.
   (i) Find the length of a side of the equilateral triangle made by Amali.
   (ii) Find the length of a side of the square made by Sandamini.

(6) The length and breadth of a rectangular shaped flower bed are 7 m and 3 m respectively. How many square shaped bricks of length 25 cm each are needed to place one row of bricks around the flower bed without any space left between the bricks?

(7) The length of a rectangular shaped playground is twice its breadth. If the perimeter of the playground is 360 m, find its length and its breadth.

2.2 Perimeter of a composite rectilinear plane figure

You have learnt that a plane figure which is composed of several plane figures is called a composite plane figure. Now let us learn how to find the perimeter of a composite plane figure which is composed of two plane figures.

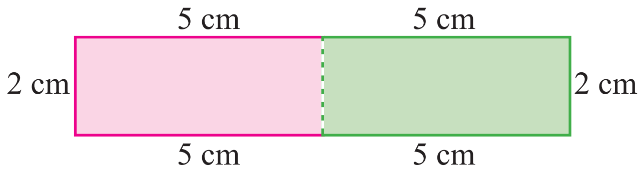
Two rectangular shaped pieces of paper which are 5 cm in length and 2 cm in breadth are given below.

The perimeter of one rectangular shaped piece of paper = 5 cm + 2 cm + 5 cm + 2 cm
= 14 cm

The sum of the perimeters of the two rectangular shaped pieces of paper = 14 cm + 14 cm
= 28 cm

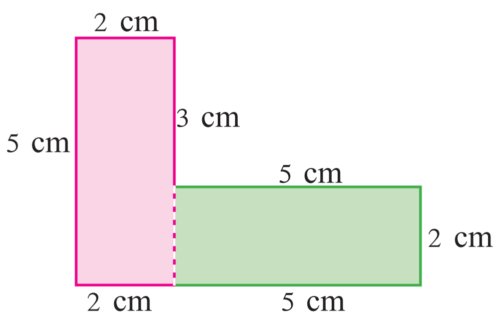
Let us find the perimeter of several composite plane figures formed with these two rectangular shaped pieces of paper.

(i)



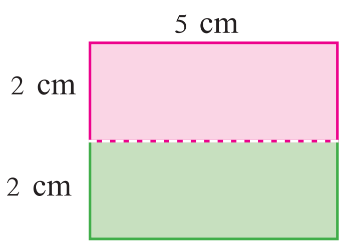
The perimeter of the figure = 5 cm + 5 cm + 2 cm + 5 cm + 5 cm + 2 cm
= 24 cm

(ii)



The perimeter of the figure = 5 cm + 2 cm + 3 cm + 5 cm + 2 cm + 5 cm + 2 cm
= 24 cm

(iii)



The perimeter of the figure = 5 cm + 2 cm + 2 cm + 5 cm + 2 cm + 2 cm
= 18 cm

It must be clear to you through these examples, that the perimeter of each of the composite plane figures formed is less than the sum of the perimeters of the two rectangles.

Hence, when calculating the perimeter of a composite rectilinear plane figure, only the lengths of all the straight line segments by which the figure is bounded should be added.

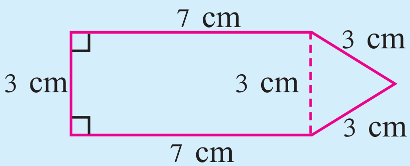
Note :

The perimeter of a composite plane figure cannot be obtained by adding together all the perimeters of the plane figures which the composite figure is composed of.

Example 1

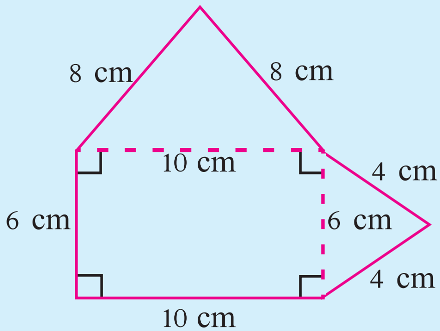
Calculate the perimeter of each of the figures given below.

(i)



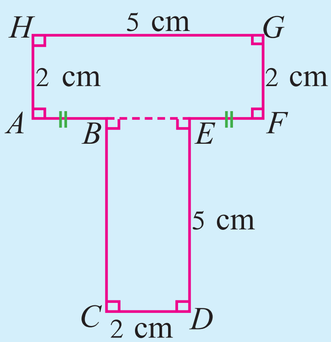
Perimeter $= 7\text{ cm} + 3\text{ cm} + 3\text{ cm} + 7\text{ cm} + 3\text{ cm}$
$= 23\text{ cm}$

(ii)



Perimeter $= 8\text{ cm} + 4\text{ cm} + 4\text{ cm} + 10\text{ cm} + 6\text{ cm} + 8\text{ cm}$
$= 40\text{ cm}$

(iii)



$GH = 5\text{ cm}$

$AB = EF$

$2\ AB = 5\text{ cm} - 2\text{ cm} = 3\text{ cm}$

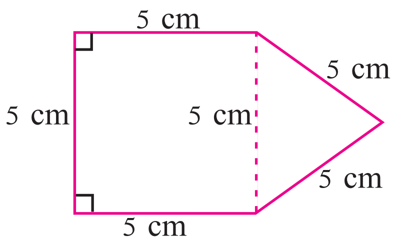
$\therefore\ AB = 1.5\text{ cm}$

Perimeter of the figure $= 5\text{ cm} + 2\text{ cm} + 1.5\text{ cm} + 5\text{ cm} + 2\text{ cm} + 5\text{ cm} + 1.5\text{ cm} + 2\text{ cm}$
$= 24\text{ cm}$

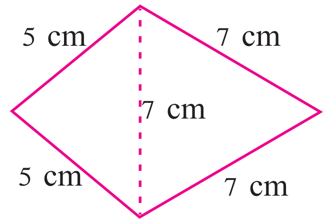
Exercise 2.1

(1) Calculate the perimeter of each of the figures given below.

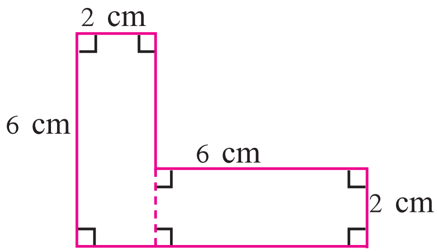
(i)



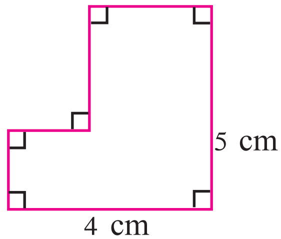
(ii)



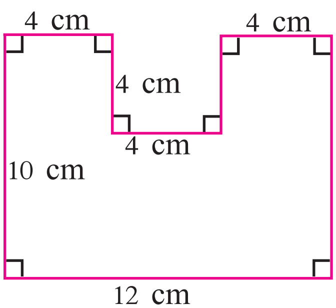
(iii)



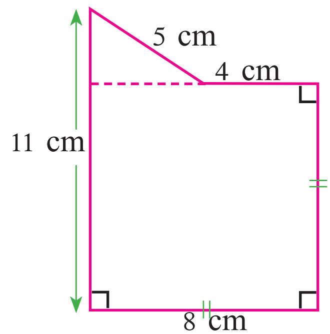
(iv)



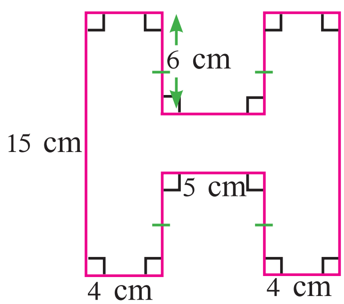
(v)



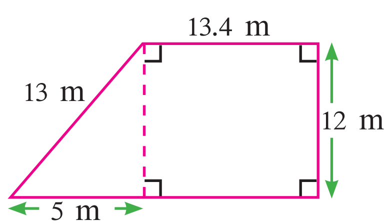
(vi)



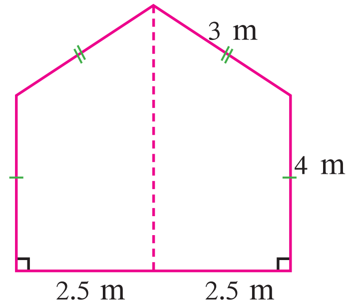
(vii)



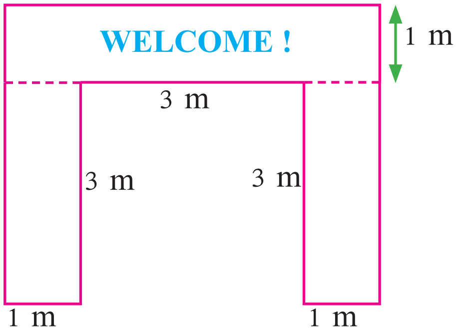
(viii)



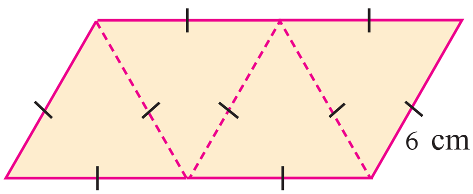
(2) A figure of a gate with two panels is given here. Calculate the perimeter of the gate.



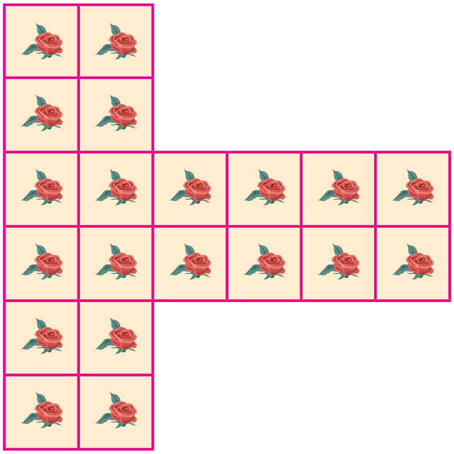
(3) A figure of an entrance structure constructed to welcome the students of grade 1 to a school is given with its measurements. Find the minimum length of the ribbon required to fix around the entrance structure.



(4) A figure of a net used to construct a solid is shown here. Calculate its perimeter.



(5) A section of a courtyard constructed with square shaped floor tiles of length 40 cm each is shown in the figure. Find the perimeter of this section?



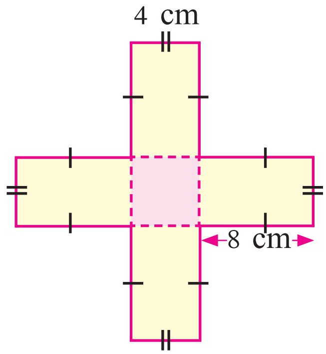
(6) If the perimeter of a wall hanging composed of a square shaped wooden lamina and an equilateral triangular shaped wooden lamina with base equal to a side of the square is 160 cm,

(i) calculate the length of a side of the square shaped wooden lamina.

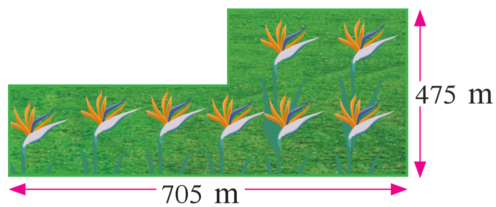
(ii) calculate the perimeter of the equilateral triangular shaped wooden lamina.

(7) What is the least of the perimeters of the composite plane figures that can be made with two rectangles of length 6 cm and breadth 4 cm each?

(8) A composite figure formed with four rectangles of length 8 cm and breadth 4 cm each and a square of side length 4 cm is shown here. Calculate the perimeter of the figure.



(9) Every morning Binuli walks twice around the park shown in the figure. Find the total distance she walks around the park each day.



Summary

- The perimeter of a composite plane figure which is composed of several plane figures is not equal to the sum of the perimeters of the plane figures of which it is composed.
- When calculating the perimeter of a composite rectilinear plane figure, only the lengths of the straight line segments by which it is bounded should be added.

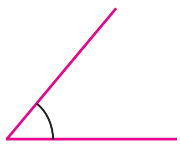
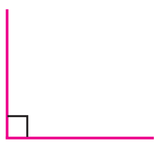
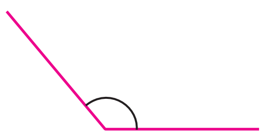
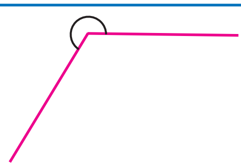
3 Angles

By studying this lesson you will be able to,

- identify pairs of complementary angles, supplementary angles, adjacent angles and vertically opposite angles,
- identify that the sum of the angles which lie around a point on one side of a straight line is 180˚,
- identify that the sum of the angles around a point on a plane is 360˚,
- identify that the vertically opposite angles created by two intersecting straight lines are equal, and
- calculate the magnitudes of angles associated with straight lines.

3.1 Angles

You have learnt in Grade 7 that the standard unit used to measure angles is **degrees** and that one degree is written as 1˚.

Angle	Figure	Note
Acute Angle		An angle of magnitude less than 90° is called an **acute angle**.
Right Angle		An angle of magnitude 90° is called a **right angle**.
Obtuse Angle		An angle of magnitude greater than 90° but less than 180° (that is, an angle of magnitude between 90° and 180°) is called an **obtuse angle**.
Straight Angle		An angle of magnitude 180° is called a **straight angle**.
Reflex Angle		An angle of magnitude between 180° and 360° is called a **reflex angle**.

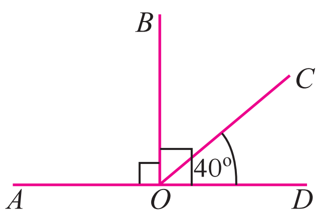
Do the review exercise to recall the above given facts which you learnt in Grade 7 under the lesson on angles.

Review Exercise

(1) Copy the two groups A and B given below and join them appropriately.

A	B
135°	Acute angle
90°	Right angle
180°	Obtuse angle
35°	Straight angle
245°	Reflex angle
190°	
280°	

(2) By considering the given figure, find the magnitude of each of the angles given below and write the type of each angle.

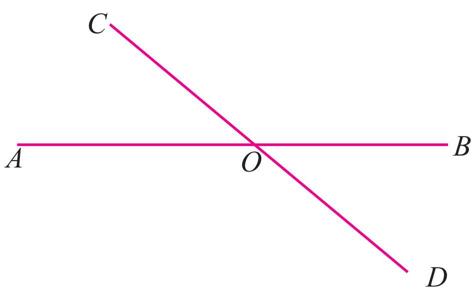


(i) $A\hat{O}B$ (ii) $C\hat{O}D$
(iii) $B\hat{O}D$ (iv) $B\hat{O}C$
(v) $A\hat{O}C$ (vi) $A\hat{O}D$

(3) Draw the following angles using a protractor and name them.

(i) $P\hat{Q}R = 60^\circ$ (ii) $A\hat{B}C = 90^\circ$ (iii) $X\hat{Y}Z = 130^\circ$ (iv) $K\hat{L}M = 48^\circ$

(4) As shown in the figure, draw two straight line segments AB and CD such that they intersect each other at O.



(i) Measure the magnitude of each of the angles $A\hat{O}C$, $C\hat{O}B$, $B\hat{O}D$ and $A\hat{O}D$ and write them down.

(ii) What is the value of $A\hat{O}C + C\hat{O}B$?

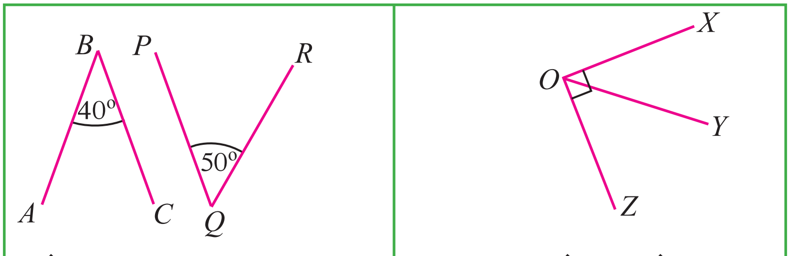
(iii) Are the two angles $A\hat{O}C$ and $B\hat{O}D$ equal to each other?

3.2 Complementary angles and supplementary angles

Let us identify what complementary angles and supplementary angles are.

• Complementary angles

Two pairs of angles are shown in the figure given below. Let us consider the sum of the magnitudes of each pair of angles.



$A\hat{B}C + P\hat{Q}R = 40^\circ + 50^\circ = 90^\circ$

$X\hat{O}Y + Y\hat{O}Z = 90^\circ$

The sum of the magnitudes of the two angles of each pair is obtained as 90°.

If the sum of a pair of acute angles is 90°, then that pair of angles is called a pair of **complementary angles**.

According to this explanation, in the figure given above,

the angles $A\hat{B}C$ and $P\hat{Q}R$ are a pair of complementary angles, and
the angles $X\hat{O}Y$ and $Y\hat{O}Z$ are a pair of complementary angles.

The acute angle which needs to be added to a given acute angle for the sum of the two angles to be 90° is called the **complement** of the given angle.

$30^\circ + 60^\circ = 90^\circ$. Hence, the complement of 30° is 60°.

Example 1

Calculate the complement of 38°.

Since $90^\circ - 38^\circ = 52^\circ$, the complement of 38° is 52°.

Example 2

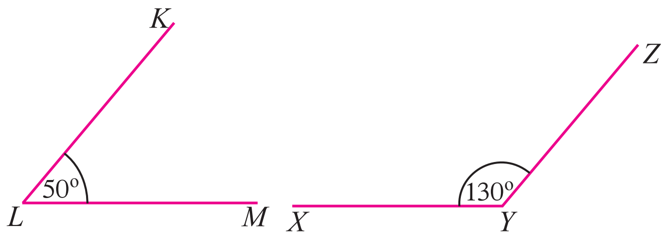
If $A\hat{B}C = 48^o$, $P\hat{Q}R = 66^o$, $K\hat{L}M = 42^o$ and $X\hat{Y}Z = 24^o$; name the pairs of complementary angles among these angles.

$48^o + 42^o = 90^o$. $\therefore$ $A\hat{B}C$ and $K\hat{L}M$ are a pair of complementary angles.
$66^o + 24^o = 90^o$. $\therefore$ $P\hat{Q}R$ and $X\hat{Y}Z$ are a pair of complementary angles.

• Supplementary angles

Let us consider the sum of the two angles given in the figure.

$K\hat{L}M + X\hat{Y}Z = 50^o + 130^o$
$= 180^o$



If the sum of a pair of angles is 180°, then that pair of angles is called a pair of **supplementary angles.**

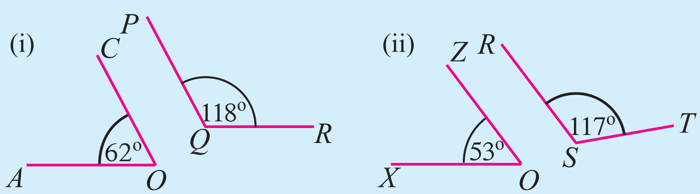
According to this explanation, $K\hat{L}M$ and $X\hat{Y}Z$ are a pair of supplementary angles.

The angle which needs to be added to a given angle of less than 180° for the sum to be 180° is called the **supplement** of the given angle.

$60^o + 120^o = 180^o$
$\therefore$ The supplement of 60^o is 120^o.

Example 3

Explain whether the pairs of angles given in the figure are supplementary angles.



(i) $A\hat{O}C + P\hat{Q}R = 62^o + 118^o$
$= 180^o$
$\therefore$ $A\hat{O}C$ and $P\hat{Q}R$ are a pair of supplementary angles.

(ii) $X\hat{O}Z + R\hat{S}T = 53^\circ + 117^\circ$

$= 170^\circ$

Since the sum of the two angles is not 180°, $X\hat{O}Z$ and $R\hat{S}T$ are not a pair of supplementary angles.

Exercise 3.1

(1) Copy and complete.

(i) The complement of 60° is ....
The supplement of 60° is ....

(ii) The complement of 75° is ....
The supplement of 75° is ....

(iii) The complement of 25° is ....
The supplement of 25° is ....

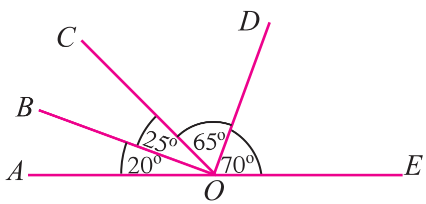
(iv) The complement of 1° is ....
The supplement of 1° is ....

(2) From among the angles $A\hat{B}C = 72^\circ$, $P\hat{Q}R = 15^\circ$, $X\hat{Y}Z = 28^\circ$, $K\hat{L}M = 165^\circ$, $B\hat{O}C = 18^\circ$, $M\hat{N}L = 108^\circ$ and $D\hat{E}F = 75^\circ$, select and write down,

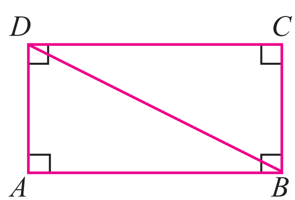
(i) two pairs of complementary angles.
(ii) two pairs of supplementary angles.

(3) According to the figure given here,

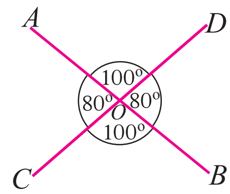
(i) what is the sum of $B\hat{O}C$ and $C\hat{O}D$?
(ii) what is the complement of $B\hat{O}C$?
(iii) what is the magnitude of $A\hat{O}D$?
(iv) what is the sum of $A\hat{O}D$ and $D\hat{O}E$?
(v) what is the supplement of $D\hat{O}E$?
(vi) what is the complement of $D\hat{O}E$?



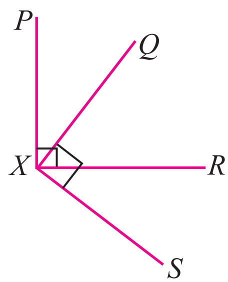
(4) (i) Write two pairs of complementary angles in the given figure.



(ii) The straight line segments AB and CD intersect at O. Write four pairs of supplementary angles in the figure.



(5) Write two pairs of complementary angles according to the information marked in the given figure.



(6) Copy these statements in your exercise book and place a ✓ in front of the correct statements and a ✗ in front of the incorrect statements.

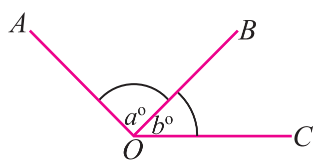
(i) The complement of an acute angle is an acute angle.

(ii) The complement of an acute angle is an obtuse angle.

(iii) The supplement an obtuse angle is an obtuse angle.

(iv) The supplement of an acute angle is an obtuse angle.

3.3 Adjacent angles



Let us consider the arms and the vertex of the two angles $A\hat{O}B$ and $B\hat{O}C$ in the figure.

The arms of $A\hat{O}B$ are AO and BO. The vertex is O.
The arms of $B\hat{O}C$ are BO and CO. The vertex is O.

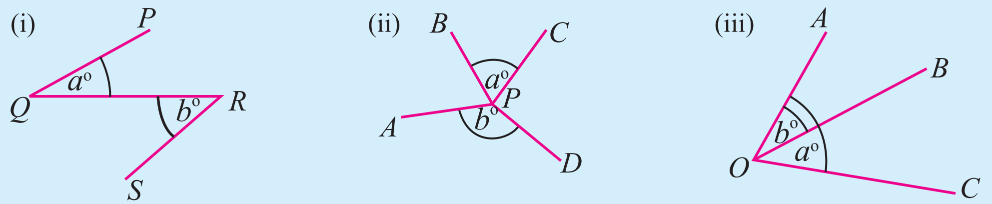
The arm BO belongs to both angles. Hence, BO is a common arm. The vertex of both angles is O. Hence, O is the **common vertex**. Moreover, these two angles are located on either side of the **common arm** OB .

A pair of angles which have a common arm and a common vertex and are located on either side of the common arm is called a pair of **adjacent angles**.

According to this explanation, $A\hat{O}B$ and $B\hat{O}C$ in the figure given above are a pair of adjacent angles.

Example 1

Explain whether the pairs of angles denoted by a and b in the figures given below are pairs of adjacent angles.



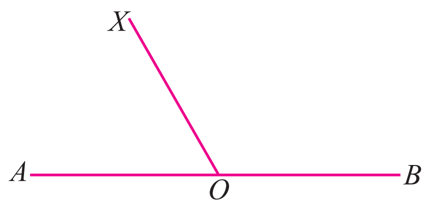
(i) QR is the common arm of both angles. The two angles are located on either side of QR. But there isn't a common vertex. Hence, $P\hat{Q}R$ and $Q\hat{R}S$ are not adjacent angles.

(ii) Both angles have a common vertex. But they do not have a common arm. Therefore, $B\hat{P}C$ and $A\hat{P}D$ are not adjacent angles.

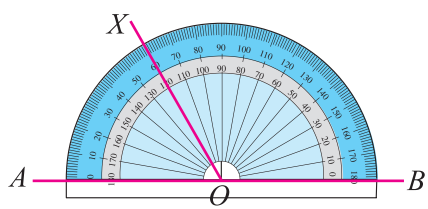
(iii) The angles $A\hat{O}B$ and $A\hat{O}C$ have a common arm and a common vertex. The common arm is AO. However, the two angles are not located on either side of the common arm. Therefore, $A\hat{O}B$ and $A\hat{O}C$ are not adjacent angles.

• Adjacent angles on a straight line

A pair of adjacent angles named $A\hat{O}X$ and $B\hat{O}X$ is created by the straight line XO meeting the straight line AB at O. Let us measure these two angles by using a protractor.



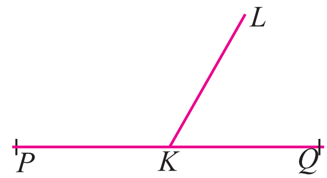
It is clear that in the figure, $A\hat{O}X = 60^{\circ}$ and $B\hat{O}X = 120^{\circ}$ (You can read the magnitudes of both angles at the same time by placing the base line of the protractor on the line AOB).



Activity 1

Step 1 - Draw a straight line segment in your exercise book and name it PQ.

Step 2 - Draw the straight line KL with the point K located on PQ.



Step 3 - Measure $P\hat{K}L$ and $Q\hat{K}L$ using the protractor and write down their magnitudes.

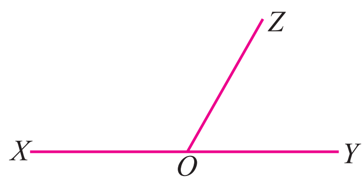
Step 4 - Fill in the blanks below the figure.

$P\hat{K}L + Q\hat{K}L =$ .... + ...

$=$ ....

Step 5 - As above, engage in the activity for another two figures, and investigate the possible conclusion that can be drawn.

The line segment XY is divided into the two line segments OX and OY by the point O located on XY.



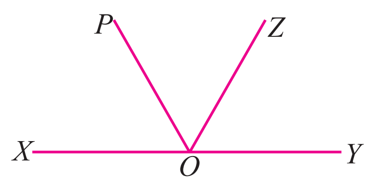
The sum of the two adjacent angles $X\hat{O}Z$ and $Z\hat{O}Y$, where OZ is the common arm, and OX and OY are the other arms, can be shown to be 180° by measuring the two angles separately.

This establishes the fact that a pair of adjacent angles, located on a straight line in this manner is a pair of supplementary angles.

Let us divide the angle $X\hat{O}Z$ into two by the straight line OP in the figure.

Then $X\hat{O}Z = X\hat{O}P + P\hat{O}Z$

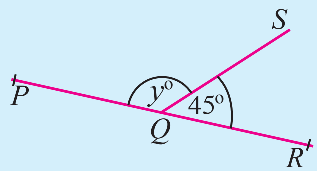
$\therefore\ X\hat{O}P + P\hat{O}Z + Z\hat{O}Y = X\hat{O}Z + Z\hat{O}Y = 180^\circ$.



The sum of the angles around a point on a straight line, located on one side of the straight line is 180°.

Example 2

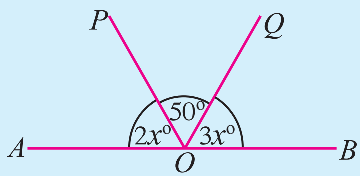
In the given figure, PR is a straight line segment. Find the magnitude of $P\hat{Q}S$.



$$y + 45 = 180$$
$$y + 45 - 45 = 180 - 45$$
$$y = 135$$
$$P\hat{Q}S = 135^o$$

Example 3

Find the magnitude of $A\hat{O}P$ according to the information marked in the figure.



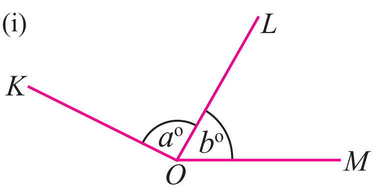
$2x + 50 + 3x = 180$ (the sum of the angles on a straight line is 180°)

$$5x + 50 = 180$$
$$5x + 50 - 50 = 180 - 50$$
$$\frac{5x}{5} = \frac{130}{5}$$
$$x = 26$$
$$\therefore\ A\hat{O}P = 2x^o = 2 \times 26^o = 52^o$$

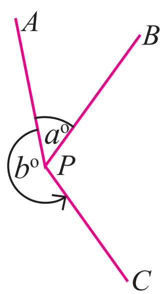
Exercise 3.2

(1) Write whether the pair of angles marked as a and b in each figure is a pair of adjacent angles.

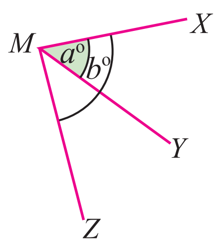
(i)



(ii)

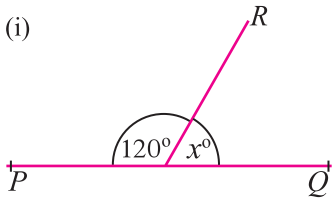


(iii)

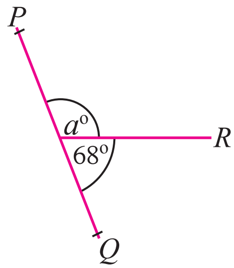


(2) If PQ is a straight line segment in each figure given below, find the magnitude of the angle marked by an English letter.

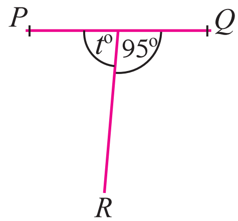
(i)



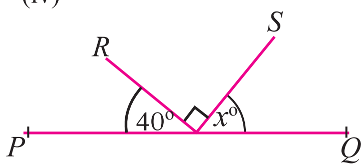
(ii)



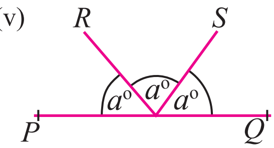
(iii)



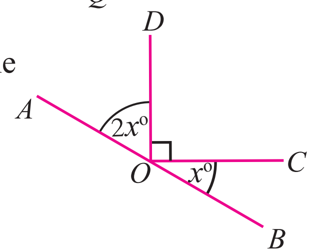
(iv)



(v)



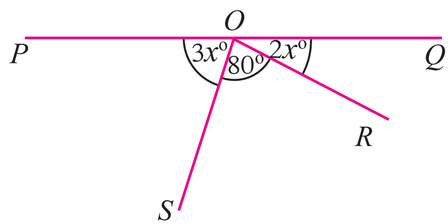
(3) In the figure, if AB is a straight line segment, find the magnitude of $A\hat{O}D$.



(4) PQ is a straight line segment. According to the information marked in the figure,

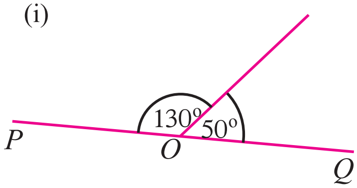
(i) find the magnitude of $P\hat{O}S$.

(ii) find the magnitude of $S\hat{O}Q$.

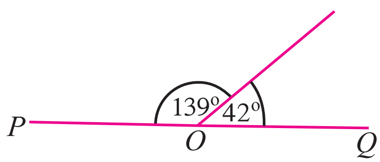


(5) Conclude whether POQ in each of the given figures is a straight line.

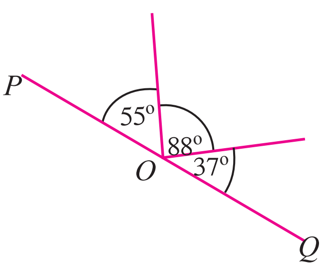
(i)



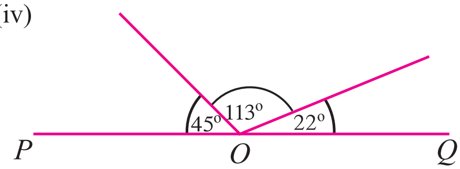
(ii)



(iii)

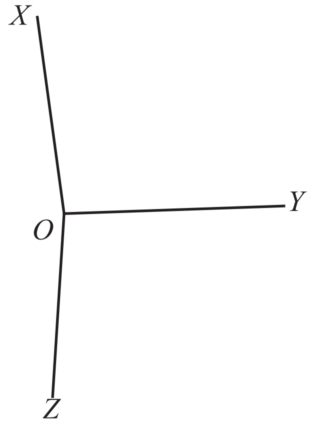


(iv)

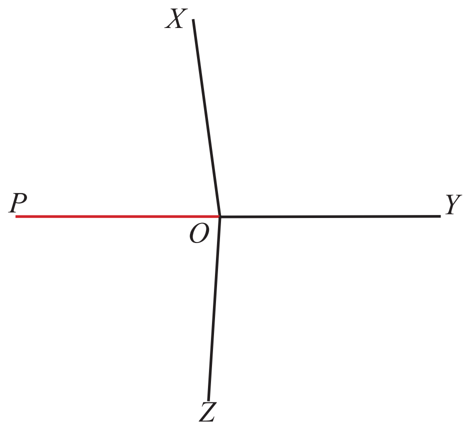


3.4 The sum of the angles around a point on a plane

Consider the angles $X\hat{O}Y$, $Y\hat{O}Z$ and $Z\hat{O}X$ located around the point O in the figure. Let us find the value of $X\hat{O}Y + Y\hat{O}Z + Z\hat{O}X$.



To do this, produce the straight line YO to P.



Method 1

Since POY is a straight line,

$$P\hat{O}X + X\hat{O}Y = 180^\circ$$
$$P\hat{O}Z + Z\hat{O}Y = 180^\circ$$
$$\therefore\ P\hat{O}X + X\hat{O}Y + P\hat{O}Z + Z\hat{O}Y = 180^\circ + 180^\circ$$
$$= 360^\circ$$

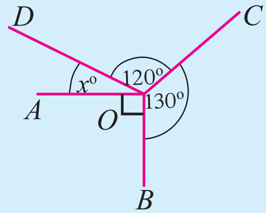
Method 2

$$Z\hat{O}X = Z\hat{O}P + P\hat{O}X$$
$$\therefore\ X\hat{O}Y + Y\hat{O}Z + Z\hat{O}X = X\hat{O}Y + Y\hat{O}Z + Z\hat{O}P + P\hat{O}X$$
$$= \underbrace{X\hat{O}Y + P\hat{O}X}_{\text{Supplementary Angles}} + \underbrace{Y\hat{O}Z + Z\hat{O}P}_{\text{Supplementary Angles}}$$
$$= 180^\circ + 180^\circ = 360^\circ$$

The sum of the angles located around a point on a plane is 360°.

Example 1

Find the magnitude of the angle marked as $A\hat{O}D$ in the given figure.

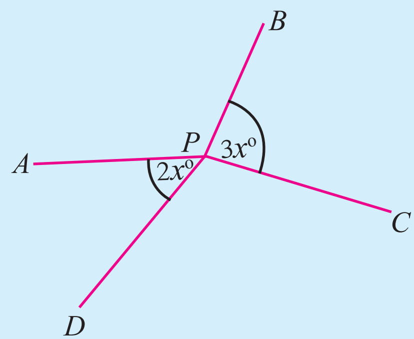


$x + 120 + 130 + 90 = 360$ (the sum of the angles around a point is 360^o)

$$x + 340 = 360$$
$$x + 340 - 340 = 360 - 340$$
$$x = 20$$
$$\therefore A\hat{O}D = 20^o$$

Example 2

If $A\hat{P}B = 150^o$ and $D\hat{P}C = 100^o$ in the figure, find the magnitude of $B\hat{P}C$.



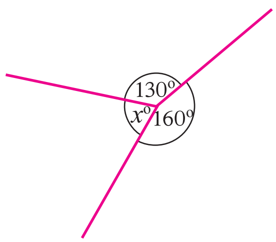
Because the sum of the angles around P is 360^o,

$$2x + 150 + 3x + 100 = 360$$
$$5x + 250 = 360$$
$$5x + 250 - 250 = 360 - 250 = 110$$
$$\frac{5x}{5} = \frac{110}{5}$$
$$x = 22$$

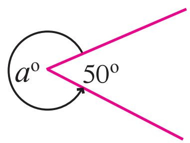
$\therefore B\hat{P}C = 3 \times 22^o = 66^o$

Exercise 3.3

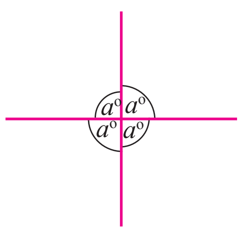
(1) Find the value of x^o.



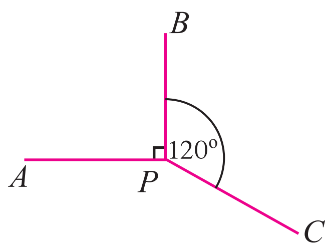
(2) Find the value of a^o.



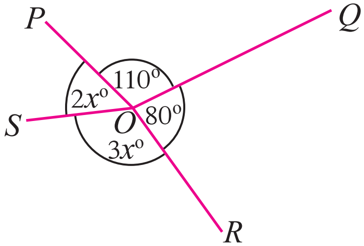
(3) Find the value of a^o.



(4) Find the magnitude of $A\hat{P}C$.

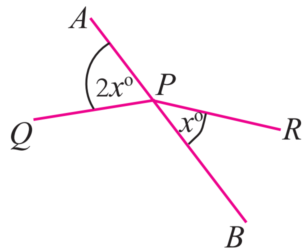


(5) Find the magnitude of $S\hat{O}R$.



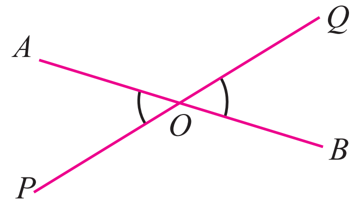
(6) AB is a straight line.

If $A\hat{P}R = 150^{o}$, find the magnitude of $Q\hat{P}B$.



3.5 Vertically opposite angles

The two straight lines AB and PQ shown in the figure intersect at point O. The two angles $A\hat{O}P$ and $B\hat{O}Q$ which are located vertically opposite each other as shown here are called **vertically opposite angles**.

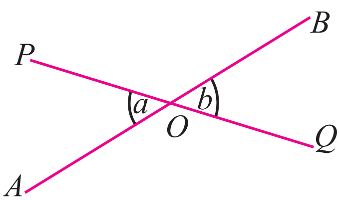


The two angles $A\hat{O}Q$ and $B\hat{O}P$ in the figure are also a pair of vertically opposite angles.

Two pairs of vertically opposite angles are always created by the intersection of two straight lines. Each pair has a common vertex and the two angles are located vertically opposite each other across the common vertex.

Activity 2

Step 1 - In your exercise book, draw two straight lines which intersect each other as shown in the figure and include the information given in the figure.



Step 2 - Copy the figure on a tissue paper and name it also as in the above figure.

Step 3 - Keep the two drawn figures such that they coincide with each other and hold them in place with a pin at point O.

Step 4 - Rotate the tissue paper half a circle around the point O and see whether the two angles a and b coincide with each other.

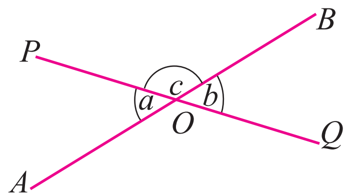
Step 5 - Engage in the activity as above for another two cases and examine whether the vertically opposite angles coincide with each other.

Investigate the conclusion that can be drawn from this activity.

It can be concluded based on the above activity, that vertically opposite angles created by the intersection of two straight lines are equal to each other.

Vertically opposite angles created by the intersection of two straight lines are equal to each other.

Let us investigate whether this is true by another method.
PQ and AB in the figure are straight line segments.



$a + c = 180^\circ$ (AB is a straight line)

$b + c = 180^\circ$ (PQ is a straight line)

$\therefore a + c = b + c$

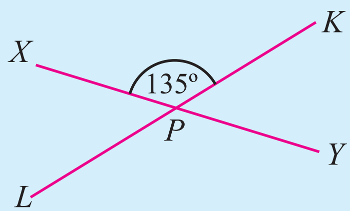
$a + c - c = b + c - c$ (subtracting c from both sides)

$\therefore a = b$

$\therefore$ The vertically opposite angles $A\hat{O}P$ and $B\hat{O}Q$ are equal to each other.

Example 1

Find the magnitude of each angle around the point P in the given figure, where XY and KL are straight line segments.



$L\hat{P}Y = X\hat{P}K$ (vertically opposite angles are equal)

$\therefore L\hat{P}Y = 135^\circ$

$X\hat{P}L + 135^\circ = 180^\circ$ (the sum of the angles on the straight line LK is 180°)

$\therefore X\hat{P}L = 180^\circ - 135^\circ$

$= 45^\circ$

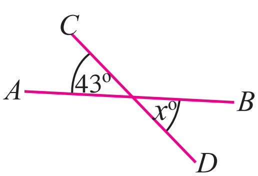
$K\hat{P}Y = X\hat{P}L$ (vertically opposite angles are equal)

$\therefore K\hat{P}Y = 45^\circ$

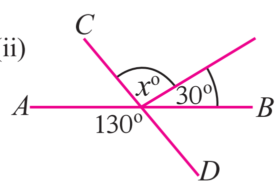
Exercise 3.4

(1) Find the magnitude of each of the angles marked by an English letter in the figures given below (AB, CD and EF are straight lines).

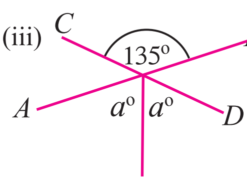
(i)



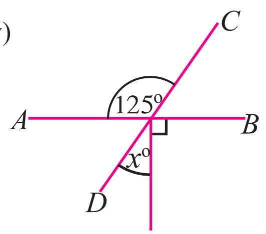
(ii)



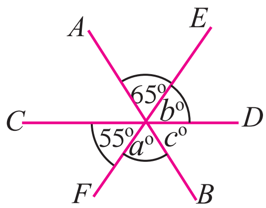
(iii)



(iv)

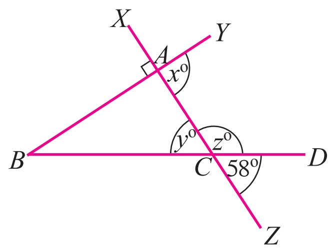


(v)



(2) (i) Find the values of the angles denoted by *x*, *y* and *z* in the given figure (*BY*, *BD* and *XZ* are straight lines).

(ii) $A\hat{B}C$ and $A\hat{C}B$ are a pair of complementary angles. What is the magnitude of $A\hat{B}C$?



Summary

- If the sum of a pair of acute angles is 90°, then that pair of angles is called a pair of complementary angles.
- The acute angle which needs to be added to a given acute angle for the sum of the two angles to be 90° is called the complement of the given angle.
- If the sum of a pair of angles is 180°, then that pair of angles is called a pair of supplementary angles.
- The angle which needs to be added to a given angle of less than 180° for the sum to be 180° is called the supplement of the given angle.
- A pair of angles which have a common arm and a common vertex and are located on either side of the common arm is called a pair of adjacent angles.
- The sum of the angles located around a point on one side of a straight line is 180°.
- The sum of the angles located around a point on a plane is 360°.
- Vertically opposite angles created by the intersection of two straight lines are equal to each other.

4

Directed Numbers

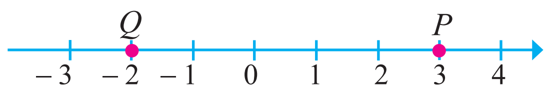
By studying this lesson you will be able to,

- subtract a directed number from another directed number, and
- multiply directed numbers and divide a directed number by a directed number.

4.1 Directed numbers

Let us recall what you learnt in Grade 7 about directed numbers.

Consider the following number line on which the points P and Q are marked.



- On this number line, the point P represents the directed number $(+3)$ and the point Q represents the directed number (-2).
- $(+3)$ is most often written as 3.
- (-2) and $(+3)$, are located on the number line in opposite directions to each other from zero.
- + (positive) sign is used to denote the direction in which the directed number $(+3)$ is located with respect to zero on the number line.
- – (negative) sign is used to denote the opposite direction, in which the directed number (-2) is located.

The **magnitude** of a number represented by a point on the number line is the distance on the number line from zero to that point.

Furthermore, a number gets its sign as + or – according to the position of that number, whether it is to the right or to the left of the point which represents 0.

- Since the distance from zero to point P is 3 units, the magnitude of the directed number $(+3)$ is 3. The magnitude of the directed number (-2) is 2.

In a directed number, the numerical value shows its magnitude and the + or – sign its direction.

$(+3)$, (-7), $(+2.5)$, (-3.4), $\left(+3\frac{1}{2}\right)$, $\left(-5\frac{1}{4}\right)$ are some examples of directed numbers.

Note

- It is important to note that, while using the symbol + or – to denote the direction of a number, the symbol + is also used to denote the addition of two directed numbers and the symbol – is used to denote subtraction of a directed number from another directed number.
- We have to understand that the symbols + and – are used in two different senses here.
- To differentiate this clearly, we write directed numbers within brackets.

• Adding directed numbers

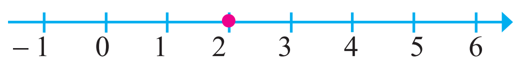
Since the sign of a directed number is important, we should pay special attention to the sign when performing mathematical operations.

You learnt in Grade 7 how the addition of directed numbers can be explained easily by using the number line.

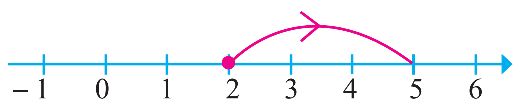
The addition of directed numbers can be explained easily by using the number line in the following manner too.

➤ Let us find the value of (+2) + (+3) by using the number line.

- Mark the directed number (+2) on the number line.

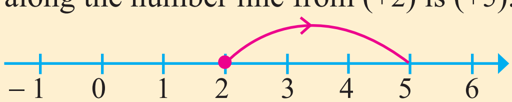


- From this point, move 3 units, which is the magnitude of (+3), to the right, which is the direction of (+3) along the number line.



- The directed number (+5) represented by the ending point is the sum of the above two directed numbers.

That is, the directed number which is obtained when you move 3 units to the right along the number line from (+2) is (+5).



∴ (+2) + (+3) = (+5)

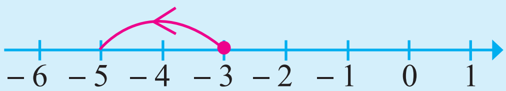
The steps we followed are given below in order.

When adding a directed number to another directed number do the following.

- Mark the point which represents the first directed number on the number line.
- From that point, move a distance **equal to the magnitude of the second directed number** towards the **direction of the second directed number**.
- The directed number which is represented by the ending point is the answer.

Example 1

Find the value of $(-3) + (-2)$ by using the number line.



From (-3), when you move two units to the left, which is the direction of (-2), the directed number you end at is (-5).

$\therefore\ (-3) + (-2) = (-5)$

Adding directed numbers without using the number line

What you learnt in Grade 7 about the addition of directed numbers without using the number line is presented here.

Let us find the value without using the number line.

When adding two directed numbers of the same sign, first add the two numbers without considering their signs. Include the same sign in the answer.

(i) $(+3) + (+2) = (+5)$

(ii) $(-4) + (-6) = (-10)$

When adding two directed numbers of different signs (positive and negative), first find the difference of their numerical values, without considering their signs. Include the sign of the directed number having the larger magnitude in the answer.

(iii) Let us find the value of $(+8) + (-3)$.

$8 - 3 = 5$

$\therefore\ (+8) + (-3) = (+5)$

(iv) Let us find the value of $(+4.2) + (-6.3)$.

$6.3 - 4.2 = 2.1$

$\therefore\ (+4.2) + (-6.3) = (-2.1)$

Do the following review exercise to recall what you learnt in Grade 7 about directed numbers.

Review Exercise

(1) Find the value of each of the following by using the number line.

(i) (+2) + (+6) (ii) (+8) + (−5) (iii) (−2) + (+3)
(iv) (−3) + (−4) (v) (+4) + (−6)

(2) Find the value of each of the following.

(i) (+2) + (+3) (ii) (−4) + (−2) (iii) (−3) + (+5)
(iv) (+4) + (−10) (v) (−7) + (+7) (vi) (+2) + (+5) + (+3)
(vii) (−3) + (−1) + (−4) (viii) (+2) + (+4) + (−9) (ix) $\left(+\frac{5}{7}\right) + \left(-\frac{2}{7}\right)$
(x) (+3.4) + (−5.2) (xi) (−8.11) + (+8.11)

4.2 Subtracting a directed number from another directed number

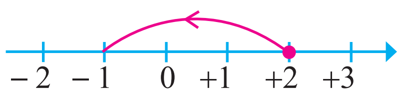
Now let us consider how to subtract a directed number from another directed number by using the number line. Let us first find out what is meant by the direction opposite to that of a given directed number.

- The magnitude of (+3) is 3 and its direction is toward the right.
  The **direction opposite** to that of (+3) is towards the left.
- The magnitude of (–3) is 3 and its direction is towards the left.
  The **direction opposite** to that of (–3) is towards the right.

➤ Let us find the value of (+2) – (+3) by using the number line.

- First mark the directed number (+2) on the number line.

- From this point, move 3 units which is the magnitude of (+3), towards the left, which is the direction opposite to that of (+3).



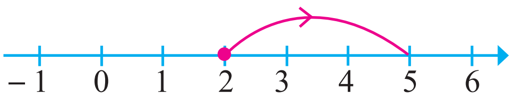
- The answer is the directed number represented by the ending point.

The answer is obtained from the point which is 3 units to the left of (+2).
∴ (+2) − (+3) = (−1)

When subtracting a directed number from a directed number, do the following.

- Mark the point which represents the first directed number on the number line.
- From this point, move a distance equal to the magnitude of the second directed number in the direction opposite to that of the second directed number.
- The directed number which is represented by the ending point is the answer.

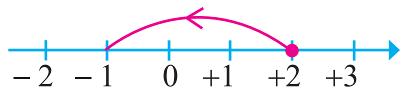
Finding the value of (+2) + (+3).



The directed number which is represented by the ending point, when you move 3 units along the number line **in the direction of (+3)** from (+2) is obtained as the answer.

∴ (+2) + (+3) = (+5)

Finding the value of (+2) – (+3).



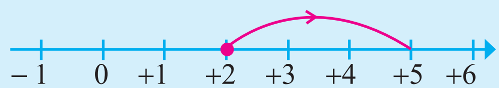
The directed number which is represented by the ending point, when you move 3 units along the number line **in the direction opposite to that of (+3)** from (+2) is obtained as the answer.

∴ (+2) – (+3) = (–1)

Example 1

Find the value of (+2) – (–3) by using the number line.

The magnitude of (–3) is 3 and the direction opposite to that of (–3) is towards the right.



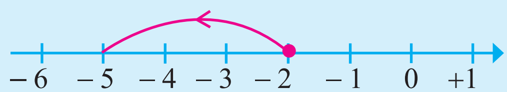
The answer is the directed number which is represented by the point located 3 units to the right of (+2).

∴ (+2) – (–3) = (+5)

Example 2

Find the value of (–2) – (+3) by using the number line.

The magnitude of (+3) is 3 and the direction opposite to that of (+3) is towards the left.



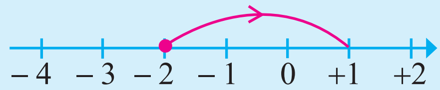
The answer is the directed number which is represented by the point located 3 units to the left of (–2).

∴ (–2) – (+3) = (–5)

Example 3

Find the value of (–2) – (–3) by using the number line.

The magnitude of (–3) is 3 and the direction opposite to that of (–3) is towards the right.



The answer is the directed number which is represented by the point located 3 units to the right of (–2).

$\therefore$ (–2) – (–3) = (+1)

Exercise 4.1

(1) Find the value by using the number line.

(i) (+4) – (+2) (ii) (+1) – (–2) (iii) (–2) – (+3)

(iv) (–1) – (–3) (v) (–6) – (–5) (vi) (+2) – (–2)

• More on subtracting a directed number from a directed number

By solving the equation $a + 1 = 0$, let us find the value of a which satisfies this equation.

The value of a cannot be 0 or a positive whole number.

Let us subtract 1 from both sides of the equation $a + 1 = 0$.

$a + 1 - 1 = 0 - 1$

$a = -1$

By taking the value of a in this equation to be (–1),

we obtain the relationship (–1) + 1 = 0.

This can also be written as 1 + (–1) = 0.

(–1) is called the **additive inverse** of (+1).

Furthermore, the additive inverse of (–1) is (+1).

- Likewise, every positive number has a corresponding additive inverse which is a negative number of equal magnitude.
- Similarly, every negative number has an additive inverse which is a positive number of equal magnitude.

The number	The additive inverse of the number
(+5)	(–5)
(–5)	(+5)
(+2)	(–2)
(–2)	(+2)
(+ 3.5)	(–3.5)
$\left(-\frac{2}{3}\right)$	$\left(+\frac{2}{3}\right)$

Now let us consider subtracting a directed number from another directed number without using the number line.

5 – 2 = 3.

Let us consider subtracting 2 from 5 by 5 and 2 as directed numbers.

Let us write the additive inverse of 2 as a directed number and add it to 5.

The additive inverse of (+ 2) is (–2).

(+5) + (–2) = 3

Subtracting a number from another number is the same as adding the additive inverse of the second number to the first number.

Hence, 5 – 2 = (+5) – (+2)
= (+5) + (–2)
= (+3)

Example 4

Find the value of (+2) – (–4).

The additive inverse of (–4) is (+4).

∴ (+2) – (–4) = (+2) + (+4)
= (+6)

Example 5

Find the value of (–5) – (+2).

The additive inverse of (+2) is (–2).

∴ (–5) – (+2) = (–5) + (–2)
= (–7)

Example 6

Find the value of (–7) – (–3).

The additive inverse of (–3) is (+3).

∴ (–7) – (–3) = (–7) + (+3)
= (–4)

Example 7

Find the value of (–12) – (–15) – (+5).

(–12) – (–15) – (+5) = (–12) + (+15) + (–5)
= (+3) + (–5)
= (–2)

Example 8

Find the value of $\left(+\frac{3}{5}\right) - \left(+\frac{1}{5}\right)$.

$$\left(+\frac{3}{5}\right) - \left(+\frac{1}{5}\right) = \left(+\frac{3}{5}\right) + \left(-\frac{1}{5}\right)$$
$$= \left(+\frac{2}{5}\right)$$

Example 9

Find the value of $\left(-5\frac{1}{2}\right) - (+2)$.

$$\left(-5\frac{1}{2}\right) - (+2) = \left(-5\frac{1}{2}\right) + (-2)$$
$$= \left(-7\frac{1}{2}\right)$$

Example 10

Find the value of $(-3.2) - (+1.4)$.

$$(-3.2) - (+1.4) = (-3.2) + (-1.4)$$
$$= (-4.6)$$

Example 11

Find the value of $(-8.4) - (-2.1)$.

$$(-8.4) - (-2.1) = (-8.4) + (+2.1)$$
$$= (-6.3)$$

Exercise 4.2

(1) Fill in each cage with the suitable directed number.

(i) $(-5) - (+3) = (-5) + \square$
$= \square$

(ii) $(-3) - (-4) = (-3) + \square$
$= \square$

(iii) $(+7) - (-1) = (+7) + \square$
$= \square$

(iv) $(+7) - (-2) = (+7) + \square$
$= \square$

(2) Find the value of each of the following.

(a) (i) $(+4) - (+1)$ (ii) $(-8) - (-2)$ (iii) $(-3) - (-7)$

(iv) $(+9) - (-6)$ (v) $(-5) - (-5)$ (vi) $0 - (+3)$

(vii) $(-11) - (+4)$ (viii) $(+2) + (-1) - (-4)$ (ix) $(-5) - (+2) - (-6)$

(x) $(+4) - (+2) - (+8)$

(b) (i) $\left(+4\frac{1}{2}\right) - (-2)$ (ii) $\left(-6\frac{1}{4}\right) - \left(-\frac{1}{4}\right)$ (iii) $(+15.7) - (-2.3)$

(iv) $(-2) - (+3.5) - (-4.1)$ (v) $\left(+3\frac{1}{2}\right) - (-2) - \left(-\frac{1}{3}\right)$

4.3 Multiplying directed numbers

Now let us consider the multiplication of two directed numbers.

➤ Let us find the value of $(+6) \times (+2)$.

- Obtain the product of the magnitudes of the two directed numbers without considering their signs.

  $6 \times 2 = 12$
- The two directed numbers are of the same sign. Therefore the answer is positive. $\therefore (+6) \times (+2) = (+12)$

➤ Let us find the value of $(-6) \times (+2)$.

- Obtain the product of the magnitudes of the two directed numbers without considering their signs.

  $6 \times 2 = 12$
- The two directed numbers are of opposite signs. Therefore, the answer is negative.

  $\therefore (-6) \times (+2) = (-12)$

When multiplying two directed numbers,
- Find the product of the magnitudes of the two directed numbers without considering their signs.
- If the two directed numbers are of the same sign, include the positive sign in the answer.
- If the two directed numbers are of opposite signs, include the negative sign in the answer.

Example 1

Simplify $(-6) \times (-2)$.

$6 \times 2 = 12$

The two directed numbers are of the same sign. Therefore the answer is positive.

$\therefore (-6) \times (-2) = (+12)$

Example 2

Simplify $(+6) \times (-2)$.

$6 \times 2 = 12$

The two directed numbers are of opposite signs. Therefore the answer is negative.

$\therefore (+6) \times (-2) = (-12)$

Example 3

Simplify the following .

(i) $(+2) \times (+5)$ (ii) $(-2) \times (+3)$ (iii) $(+5) \times (-3)$ (iv) $(-4) \times (-3) \times (+2)$

(i) $(+2) \times (+5) = (+10)$

(ii) $(-2) \times (+3) = (-6)$

(iii) $(+5) \times (-3) = (-15)$

(iv) $(-4) \times (-3) \times (+2) = (+12) \times (+2) = (+24)$

Example 4

Simplify $(+2.5) \times (-5)$.

$2.5 \times 5 = 12.5$

$\therefore (+2.5) \times (-5) = (-12.5)$

Example 5

Simplify $(-3.4) \times (-12)$.

$3.4 \times 12 = 40.8$

$\therefore (-3.4) \times (-12) = (+40.8)$

Exercise 4.3

(1) Find the value.

(i) $(+5) \times (+4)$ (ii) $(-5) \times (+4)$ (iii) $(-10) \times (-5)$

(iv) $(+7) \times (-3)$ (v) $(-1) \times (-4)$ (vi) $(+11) \times 0$

(vii) $(-6) \times (+4)$ (viii) $(+12) \times (-3)$ (ix) $(-2) \times (+2) \times (-5)$

(x) $(-3) \times (-1) \times (+2) \times (-5)$ (xi) $(+2.5) \times (+2)$ (xii) $(+4.1) \times (-23)$

4.4 Dividing a directed number by a directed number

➤ Let us find the value of $(+6) \div (+2)$.

- Let us divide the two directed numbers by considering their magnitudes only, without considering their signs.

  $6 \div 2 = 3$

- The two directed numbers are of the same sign. Therefore the answer is positive.

  $\therefore (+6) \div (+2) = (+3)$

➤ Let us find the value of $(-6) \div (+2)$.

- Let us divide the two directed numbers by considering their magnitudes only, without considering their signs.

  $6 \div 2 = 3$
- The two directed numbers are of opposite signs. Therefore, the answer is negative.

  $\therefore (-6) \div (+2) = (-3)$

When dividing a directed number by another directed number,
- Divide by considering their magnitudes, without considering their signs.
- Include the positive sign in the answer, if the two directed numbers are of the same sign.
- Include the negative sign in the answer, if the two directed numbers are of opposite signs.

Example 1

Simplify $(-6) \div (-2)$.

$6 \div 2 = 3$

The two directed numbers are of the same sign. Therefore, the answer is positive.

$(-6) \div (-2) = (+3)$

Example 2

Simplify $(+6) \div (-2)$.

$6 \div 2 = 3$

The two directed numbers are of opposite signs. Therefore, the answer is negative.

$\therefore (+6) \div (-2) = (-3)$

Example 3

Simplify the following.

(i) $(+15) \div (+5)$ (ii) $(-9) \div (+3)$ (iii) $(+15) \div (-3)$ (iv) $(-9) \div (-3)$

(i) $(+15) \div (+5) = (+3)$ (ii) $(-9) \div (+3) = (-3)$

(iii) $(+15) \div (-3) = (-5)$ (iv) $(-9) \div (-3) = (+3)$

Exercise 4.4

(1) Find the value of each of the following.

(i) $(+10) \div (+2)$ (ii) $(-12) \div (-4)$ (iii) $(+15) \div (-3)$

(iv) $(-21) \div (+7)$ (v) $(-5) \div (+5)$ (vi) $\dfrac{(-20)}{(-4)}$

(vii) $\dfrac{(+2) \times (+8)}{(-4)}$ (viii) $\dfrac{(-36)}{(-6) \times (-2)}$ (ix) $\dfrac{(+5) \times (-4)}{(-2) \times (-2)}$

(x) $\dfrac{(-9) \times (-8)}{(-4) \times (+3)}$

(2) Fill in each cage with the suitable directed number.

(i) $(-20) \div \square = (-10)$ (ii) $(+18) \div \square = (-6)$ (iii) $\square \div (-2) = (+5)$

(iv) $(+4) \div \square = (-4)$ (v) $\dfrac{(+3) \times \square}{(-2)} = (+6)$ (vi) $\dfrac{\square \times (+7)}{(+2) \times \square} = \dfrac{(-28)}{\square} = (+7)$

Summary

- Subtracting a number from another number is the same as adding the additive inverse of the second number to the first number.
- A positive number is obtained, when two directed numbers of the same sign are multiplied or divided.
- A negative number is obtained when two directed numbers of opposite signs are multiplied or divided.

5

Algebraic Expressions

By studying this lesson, you will be able to,

- construct algebraic expressions with three unknown terms,
- multiply an algebraic expression by a number and by an algebraic term,
- simplify algebraic expressions, and
- find the value of an algebraic expression by substituting integers for the unknown terms.

5.1 Algebraic expressions

Let us recall what you learnt in Grade 7 about algebraic expressions.

A certain shop purchases the same amount of milk every day to sell. If we don't know the exact amount, we cannot represent it by a number, although the amount is a constant value.

As in the above situation, when the numerical value of a constant amount is not known, it is called an **unknown constant**.

The daily income of Nimal's shop takes different values depending on its daily sales.

Since the daily income of Nimal's shop is not a fixed value, it is a **variable**.

Simple letters of the English alphabet are used to represent unknown constants and variables.

Let us denote the daily income from Nimal's shop by x. Nimal gives Rs.500 to his mother daily from the income from his shop. After giving Rs.500 to his mother, Nimal has an amount of Rs. $x - 500$ remaining.

$x - 500$ is an **algebraic expression**. x and 500 are the **terms of the expression**.

If 350 rambutans are sold at Rs. x each, the income is Rs. $350x$. In the algebraic term $350x$, 350 is called the **coefficient** of x.

Do the review exercise to recall the above facts that you learnt about algebraic expressions in grade 7.

Review Exercise

(1) Complete the table given below.

Algebraic expression	Unknown term of the algebraic expression	Coefficient of the unknown term	Terms of the algebraic expression	Mathematical operations in the order they appear in the algebraic expression
$500 + 3x$	x	3	$500, 3x$	$+ , \times$
$2y + 4$				
$4p - 100$				
$p - 10$				
$3n - 7$				

(2) The length of a table is 2 meters more than its breadth.

(i) Write an algebraic expression for the length of the table by taking its breadth as b meters.

(ii) Write an algebraic expression for the breadth of the table by taking its length as a meters.

(3) A pencil, a pen and an eraser are bought for Rs. a, Rs. b and Rs. 4 respectively.

(i) Write an algebraic expression for the total amount of money needed to buy these three items.

(ii) Write an algebraic expression for the amount of money needed to buy 2 such pencils, 3 such pens and 4 such erasers.

(4) A taxi service charges Rs.100 as an initial fee and Rs.50 for each kilometer travelled. Write an algebraic expression for the total amount that has to be paid for a journey of x meters.

(5) The price of 1 kg of rice is Rs. x and the price of 1 kg of wheat flour is Rs. y.

(i) Write an algebraic expression for the total amount of money required to buy 1 kg of each type.

(ii) Write an algebraic expression for the amount of money required to buy 5 kg of rice and 2 kg of wheat flour.

(iii) Write an algebraic expression for the amount of money required to buy 500 g of each type.

(6) Simplify the algebraic expressions given below.

(a) (i) $a + a + a$ (ii) $4x + 3x$

(iii) $p + 4p - 2p$ (iv) $8a - 5a - a$

(v) $a + 2 + 2a + 3$ (vi) $6x + 10 - 4x + 7$

(b) (i) $3a + 4b + a - 3a + 5$ (ii) $5x - 3y - 4x - 2y$

(iii) $4m - 3n - 4m - n + 8$ (iv) $6x + 7y - 8 - 5x + y - 2$

(v) $2p + 3q + 4r + p - 2q - 3r$

5.2 Constructing algebraic expressions with three unknown terms

In Grade 7 we learnt to construct algebraic expressions with one or two unknown terms. Now let us consider how to construct algebraic expressions with three unknown terms.

- Let us express the total price of 10 books which cost Rs. x each, 3 pens which cost Rs. y each and 5 pencils which cost Rs. z each by an algebraic expression.

Price of the10 books = Rs. $x \times 10$ = Rs. $10x$

Price of the 3 pens = Rs. $y \times 3$ = Rs. $3y$

Price of the 5 pencils = Rs. $z \times 5$ = Rs. $5z$

The price of 10 books, 3 pens and 5 pencils = Rs. $10x + 3y + 5z$

- A cake is made with 500 g of sugar, 1 kg of wheat flour and 500 g of butter. The price of 1 kg of sugar is Rs. x, the price of 1 kg of wheat flour is Rs. y and the price of 1 kg of butter is Rs. z. Let us represent the amount of money required to purchase the items for the cake by an algebraic expression.

Price of 500 g of sugar of which 1 kg is Rs. x = Rs. $\frac{x}{2}$

Price of 1 kg of wheat flour of which 1 kg is Rs. y = Rs. y

Price of 500 g of butter of which 1 kg is Rs. z = Rs. $\frac{z}{2}$

The total amount required = Rs. $\frac{x}{2} + y + \frac{z}{2}$

Example 1

A bus depot uses x number of buses on route 1, y number of buses on route 2, z number of buses on the highway and 12 buses for school services each day. Write an algebraic expression for the total number of buses scheduled to run in a day.

Total number of buses scheduled for route 1, route 2, the highway and school services $\} = x + y + z + 12$

Example 2

Naveen gave Rs. 500 to the shop keeper to buy 2 kg of rice of which 1 kg is Rs. x, 500 g of sugar of which 1 kg is Rs.y and 250 g of flour of which 1 kg is Rs. z. Write an algebraic expression for the balance Naveen received.

Price of 2 kg of rice of which 1 kg is Rs. x = Rs. $2x$

Price of 500 g of sugar of which 1 kg is Rs. y = Rs. $\frac{y}{2}$

Price of 250 g of flour of which 1 kg is Rs z = Rs. $\frac{z}{4}$

Price of 2 kg of rice, 500 g of sugar and 250 g of flour = Rs. $2x + \frac{y}{2} + \frac{z}{4}$

The amount Naveen gave = Rs. 500

Balance Naveen received = Rs. $500 - (2x + \frac{y}{2} + \frac{z}{4})$

Exercise 5.1

(1) There are three members in a family. The ages of the mother, the father and the son are given in years by x, y, and z respectively. Using this information, construct algebraic expressions for;

(i) the sum of their ages.

(ii) the sum of their ages after 5 years.

(iii) the difference between the ages of the father and the son.

(iv) the sum of the ages of the mother and the father when the son was born.

(2) The price of a newspaper was Rs. p. If the price increased by Rs. 5, construct algebraic expressions for,

(i) the new price of the newspaper.

(ii) the price of two newspapers after the increase in price.

(iii) the profit gained from a newspaper with the new price, if the cost of printing a newspaper is Rs. q.

(iv) the profit gained from 10 copies, if Rs. r is spent for the distribution of each copy, in addition to the printing cost.

(3) v liters of water is stored in a tank. p liters of water flows out and q liters of water flows into the tank per hour. Construct an algebraic expression for the volume of water in the tank after 3 hours.

(4) There are 700 seats in an auditorium. x number of first class tickets which are Rs.1000 each, y number of second class tickets which are Rs. 500 each and z number of third class tickets which are Rs. 300 each were issued for a drama. Construct algebraic expressions for,

(i) the total number of tickets issued.

(ii) the number of seats which are not occupied.

(iii) the income from the issued tickets.

(iv) the remaining amount when half the income generated from the issued tickets and Rs. 100,000 is paid to the producer of the drama.

5.3 Multiplying an algebraic expression by a number

• Multiplying an algebraic expression by a positive number

Gift parcels are to be prepared for the students in a class. Each parcel is to contain x number of books and y number of pens. Let us find the total number of books and pens needed for 8 such parcels.

Method I

Number of books and pens in a parcel = $x + y$

Number of books and pens needed for 8 such parcels = $(x + y) \times 8$

$(x + y) \times 8$ is also written as $8\ (x + y)$.

Method II

$$
\begin{aligned}
\text{Number of books in a parcel} &= x \\
\text{Number of books needed for 8 such parcels} &= x \times 8 \\
&= 8x \\
\text{Number of pens in a parcel} &= y \\
\text{Number of pens needed for 8 such parcels} &= 8 \times y \\
&= 8y \\
\text{The number of books and pens needed for 8 parcels} &= 8x + 8y
\end{aligned}
$$

From this it is clear that $8(x + y) = 8x + 8y$.

$$\therefore\ 8(x + y) = 8x + 8y$$

- The total mass of a container packed with balls is x kg. The mass of the empty container is y kg. Let us find the total mass of the balls in 5 such containers.

Method I

$$
\begin{aligned}
\text{The mass of the balls in one container} &= x - y \\
\text{The mass of the balls in 5 such containers} &= 5(x - y)
\end{aligned}
$$

Method II

$$
\begin{aligned}
\text{The mass of the 5 containers with balls} &= 5x \\
\text{The mass of the 5 empty containers} &= 5y \\
\text{The mass of the balls in the 5 containers} &= 5x - 5y
\end{aligned}
$$

It is clear that $5(x - y) = 5x - 5y$

$$\therefore\ 5(x - y) = 5x - 5y$$

When multiplying an algebraic expression by a number, each term in the algebraic expression is multiplied by that number.

Example 1

Simplify.

(i) $2(a+b)$ (ii) $3(3x+y)$ (iii) $3(4x-7)$ (iv) $8(8y-7x+q)$

(i) $2(a+b) = 2\times a + 2\times b$
$= 2a + 2b$

(ii) $3(3x+y) = 3\times 3x + 3\times y$
$= 9x + 3y$

(iii) $3(4x-7) = 3\times 4x - 3\times 7$
$= 12x - 21$

(iv) $8(8y-7x+q) = 64y - 56x + 8q$

Exercise 5.2

(1) Simplify

(i) $5(a+4)$ (ii) $7(x+5)$ (iii) $6(2x+4)$
(iv) $4(4c+7)$ (v) $5(y-2)$ (vi) $3(3-x)$
(vii) $2(m+n-2p)$ (viii) $4(x-y+7)$ (ix) $2(x-2y-q)$

(2) Fill in the blanks.

(i) $2(x+7) = 2x + ...$ (ii) $5(6+a) = 30 + ...$ (iii) $8(4-y) = 32 - .....$
(iv) $6(x-y) = ... - 6y$ (v) $3(x-2y+z-5) = ... - 6y + ... - .....$

(3) The daily wages of a person is Rs. x and overtime payment for an hour is Rs. y. If he did 2 hours of overtime on each of the 5 days he worked,

(i) write an algebraic expression for his salary for the 5 days with overtime payments.

(ii) Due to a loan he has taken, Rs. 150 is deducted from his daily wages. Construct an algebraic expression for the amount he receives in hand for the 5 days and simplify it.

(4) A teacher bought three gift parcels for three students who came first in the third term test. Each parcel contained 5 books and 2 pens.

(i) Write an algebraic expression for the price of one such parcel by taking the price of a book as Rs. a and the price of a pen as Rs. b.

(ii) Write the total price of all three gift parcels as an algebraic expression and simplify it.

(5) On a packet of tea, the mass of the tea is mentioned as p grammes and the mass of the packet as q grammes.

(i) Obtain an algebraic expression for the mass of 20 such packets and simplify it.

(ii) The above 20 packets are packed in a box which is of mass t grammes. Obtain an algebraic expression for 12 such boxes and simplify it.

• Multiplying an algebraic expression by a negative number

When multiplying an algebraic expression by a negative number such as -2 or -1 we have to consider it as a directed number and multiply each term of the algebraic expression by it.

Example 2

Simplify

(i) $-2\ (a + 6)$ (ii) $-5\ (6 - x)$

(iii) $-\ (2m - 3n)$ (iv) $-4\ (2x + 3y - 2z)$

(i) $-2\ (a + 6) = (-2) \times a + (-2) \times 6$
$= -2a - 12$

(ii) $-5\ (6 - x) = (-5) \times 6 - (-5) \times x$
$= -30 + 5x$

(iii) $-\ (2m - 3n) = (-1) \times 2m - (-1) \times 3n$
$= -2m - (-3)n$
$= -2m + 3n$

(iv) $-4\ (2x + 3y - 2z) = (-4) \times 2x + (-4) \times 3y - (-4) \times 2z$
$= -8x + (-12y) - (-8z)$
$= -8x - 12y + 8z$

Exercise 5.3

(1) Simplify.

(i) $-3\ (x + 5)$ (ii) $-2\ (2x + 1)$ (iii) $-2\ (4 + x)$
(iv) $-6\ (a - 6)$ (v) $-\ (x + 5)$ (vi) $-\ (x - 3)$
(vii) $-2\ (8 + x + y)$ (viii) $-6\ (3b - 2 + 3a)$ (ix) $-\ (a - c - 3x)$
(x) $-3\ (6 - 2x + 3b)$

(2) Fill in the blanks.

(i) $-3\ (x + 4) = -3x - \ldots\ldots..$

(ii) $-3\ (x - 4) = -3x + \ldots\ldots..$

(iii) $-2\ (y + 2) = -2y - \ldots\ldots..$

(iv) $-2\ (y - 2) = -2y + \ldots\ldots..$

(v) $-\ (m + 2) = \ldots\ldots.. - 2$

(vi) $-\ (m - 2) = \ldots\ldots.. + 2$

(vii) $-4\ (2x + 3) = \ldots\ldots.. -12$

(viii) $-4\ (2x - 3y + 1) = \ldots\ldots.. + 12y - \ldots\ldots.$

(3) Jayamini buys x number of coconuts at Rs. 35 each and y number of mangoes at Rs. 58 each. She gives Rs. 1000 to the vendor. Construct an algebraic expression for the balance she receives and simplify it.

5.4 Multiplying an algebraic term by another algebraic term

Now let us consider multiplying an algebraic term by another algebraic term.

Let us simplify the product of the algebraic terms $5x$ and $3a$.

$$\begin{aligned}(5x) \times (3a) &= 5x \times 3a\\ &= 5 \times x \times 3 \times a\\ &= 5 \times 3 \times x \times a\\ &= 15xa\end{aligned}$$

Similarly, $2p \times 5c = 2 \times p \times 5 \times c = 2 \times 5 \times p \times c = 10pc$

$8r \times 3y = 8 \times r \times 3 \times y = 8 \times 3 \times r \times y = 24ry$

Accordingly, in the algebraic term we get by multiplying an algebraic term by another algebraic term,

- the coefficient is the product of the coefficients of the original two algebraic terms and,
- the product of the unknowns is the product of the two unknowns in the original algebraic terms.

Example 1

Simplify.

(i) $4m \times 3n$ (ii) $8k \times 5y$ (iii) $x \times 5y$

(iv) $2y \times (-2y)$ (v) $2m \times (-7xy)$ (vi) $(-2x) \times 7yz \times 2a$

(i) $4m \times 3n = (4 \times 3) \times (m \times n) = 12mn$

(ii) $8k \times 5y = (8 \times 5) \times (k \times y) = 40ky$

(iii) $x \times 5y = (1 \times 5) \times (x \times y) = 5xy$

(iv) $2y \times (-2y) = (2 \times -2) \times (y \times y) = -4y^2$

(v) $2m \times (-7xy) = (2 \times -7) \times (m \times xy) = -14mxy$

(vi) $(-2x) \times 7yz \times 2a = (-2 \times 7 \times 2) \times (x \times yz \times a) = -28axyz$

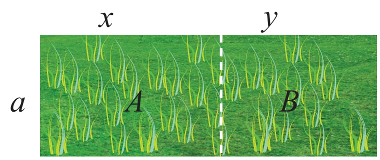
Exercise 5.4

(1) Simplify.

(i) $a \times 2b$ (ii) $2a \times 3b$ (iii) $a \times (-2b)$
(iv) $(-3a) \times 2b$ (v) $(-3x) \times (-4y)$ (vi) $(-5k) \times (-2k)$
(vii) $4p \times (-r)$ (viii) $4y \times (-3y)$ (ix) $ab \times c \times (-4x)$

5.5 Multiplying an algebraic expression by an algebraic term

A rectangular land is divided into two blocks *A* and *B* as shown in the figure. Both blocks are rectangular in shape and equal in breadth. Let us find the area of the whole land.



Method I

Area of block $A = a \times x = ax$

Area of block $B = a \times y = ay$

So the area of the whole land $= ax + ay$

We can obtain the area of the land in the following manner too.

Method II

The length of the whole land $= (x + y)$

The breadth of the land $= a$

$\therefore$ the area of the land $= a\,(x + y)$

Now it is clear that $a\,(x + y) = ax + ay$

$$\therefore\ a\,(x + y) = ax + ay$$

When multiplying an algebraic expression by an algebraic term, every algebraic term of the algebraic expression is multiplied by the given algebraic term.

Example 1

Simplify.

(i) $y(3x+5)$ (ii) $2y\,(3x+5)$ (iii) $(-y)\,(3x+5)$

(iv) $(-2y)\,(3x+5)$ (v) $2y\,(5y-3x)$

(i) $y\,(3x+5) = y \times 3x + y \times 5$

$= 3 \times y \times x + 5 \times y$

$= 3xy + 5y$

(ii) $2y\,(3x+5) = 2y \times 3x + 2y \times 5$

$= 2 \times 3 \times y \times x + 2 \times 5 \times y$

$= 6xy + 10y$

(iii) $(-y)\,(3x+5) = (-y) \times 3x + (-y) \times 5$

$= (-1) \times 3 \times y \times x + (-1) \times 5 \times y$

$= -3xy - 5y$

(iv) $(-2y)\,(3x+5) = (-2y) \times 3x + (-2y) \times 5$

$= (-2) \times 3 \times y \times x + (-2) \times 5 \times y$

$= -6xy - 10y$

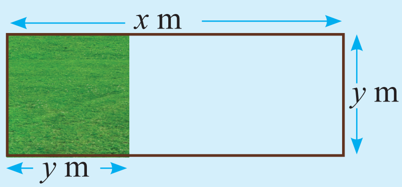
(v) $2y\,(5y-3x) = 2y \times 5y - 2y \times 3x$

$= 2 \times 5 \times y \times y - 2 \times 3 \times x \times y$

$= 10y^2 - 6xy$

Example 2

The length of a playground is x meters and its breadth is y meters. Grass is grown on one side, in a square shaped section of side length y meters. Express the area of the remaining land by an algebraic expression and simplify it.



Length of the remaining land $= x - y$

Breadth of the remaining land $= y$

Area of the remaining land $= (x-y)\,y$

$= x \times y - y \times y$

$= xy - y^2$

Exercise 5.5

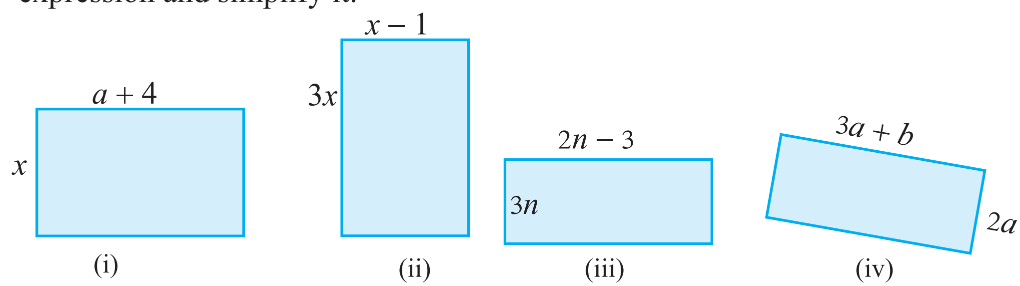
(1) Simplify.

(i) $3x\,(2y + 1)$ (ii) $3x(2y - 1)$ (iii) $3q(4p - 7)$

(iv) $(-3q)\,(4p + 8)$ (v) $2x(4p + 5y)$ (vi) $2p(4p + 5y)$

(vii) $2q(xq - z)$ (viii) $(-2q)\,(x - 4zq)$

(2) Express the area of each rectangular figure given below by an algebraic expression and simplify it.



5.6 Sum of two algebraic expressions

• Like terms

In Grade 7 you learnt that algebraic terms such as x and $2x$ with the same unknown are called **like terms.**

In each of the two terms $3xy$ and $5xy$, the coefficient is multiplied by the common term xy which is the product of the two unknowns x and y. Therefore, they are also known as **like terms**.

• Unlike terms

You learnt in Grade 7 that terms such as $2x$ and $4y$ which have different unknowns are called **unlike terms**.

Let us consider the algebraic terms $3x^2y$ and $5xy^2$.

The coefficient of $3x^2y$ is 3 and the product of the unknowns by which it is multiplied is x^2y.

The coefficient of $5xy^2$ is 5 and the product of the unknowns by which it is multiplied is xy^2.

In these two terms, the products of the unknowns are not the same.

Such algebraic terms are not **like terms**. Therefore they are known as **unlike terms**.
Like terms can be added or subtracted and simplified to a single algebraic term.

Example 1

Add $6t + 5$ and $2t + y + 3$ and simplify your answer.

$6t + 5 + 2t + y + 3 = 6t + 2t + y + 5 + 3$

$= 8t + y + 8$

Example 2

Simplify.

(i) $(2x - y + 8) + 2(3y - 10)$ (ii) $(7a - 4b + 2bc) + 2b(4a - 2c + 5)$

(i) $(2x - y + 8) + 2(3y - 10) = 2x - y + 8 + 6y - 20$

$= 2x + 5y - 12$

(ii) $(7a - 4b + 2bc) + 2b(4a - 2c + 5) = 7a - 4b + 2bc + 8ab - 4bc + 10b$

$= 7a + 6b - 2bc + 8ab$

Exercise 5.6

(1) Simplify.

(i) $3(a + 5b) + a(a + 4)$

(ii) $y(10 - y) + 3(y - 2)$

(iii) $2(8a - 5b) + 3(5a - 12)$

(iv) $3(y - 3) + (8 - 6y + x)$

(v) $a(a - 2b) + b(b + 2a - c)$

(vi) $5(x - y + z) + (4x + 3y)$

5.7 Simplifying the difference of two algebraic expression

Now let us subtract an algebraic expression from another algebraic expression and simplify it.

Let us subtract $(a + 6)$ from $(2a + 7)$.

$$(2a + 7) - (a + 6) = 2a + 7 + (-1) \times (a + 6)$$
$$= 2a + 7 + (-1) \times a + (-1) \times 6$$
$$= 2a + 7 + (-a) + (-6)$$
$$= 2a + 7 - a - 6$$
$$= 2a - a + 7 - 6$$
$$= a + 1$$

Here, the answer is obtained by multiplying each the terms of the algebraic expression which is to be subtracted by (-1) and adding them to the first algebraic expression.

Example 1

Simplify.

(i) $(4x + 3) - (2x - 3)$ (ii) $(3x + 7y) - (2x - 3y - z)$

(iii) $(10a - 8b + c) - 2\ (4a + b)$ (iv) $a\ (3a + 1) - a(a - 5)$

(i) $(4x + 3) - (2x - 3) = 4x + 3 + (-1) \times (2x - 3)$; [multiplying $(2x - 3)$ by (-1)]

$= 4x + 3 + (-1) \times 2x + (-1) \times (-3)$

$= 4x + 3 + (-2x) + 3$

$= 4x + 3 - 2x + 3$

$= 4x - 2x + 3 + 3$

$= 2x + 6$

(ii) $(3x + 7y) - (2x - 3y - z) = 3x + 7y - 2x + 3y + z$; [multiplying $(2x - 3y - z)$ by (-1)]

$= 3x - 2x + 7y + 3y + z$

$= x + 10y + z$

(iii) $(10a - 8b + c) - 2\ (4a + b) = 10a - 8b + c - 8a - 2b$; [multiplying $(4a + b)$ by (-2)]

$= 10a - 8a - 8b - 2b + c$

$= 2a - 10b + c$

(iv) $a(3a + 1) - a(a - 5) = a \times 3a + a \times 1 - a \times a + a \times 5$

$= 3a^2 + a - a^2 + 5a$

$= 2a^2 + 6a$

Exercise 5.7

(1) Simplify.

(i) $4\ (x + 2) - 2\ (x + 2)$ (ii) $4\ (x - 6) - 6\ (2 + x)$

(iii) $3\ (x - 2) - (x + 2)$ (iv) $4\ (y - 5x) - 2\ (y + 3x + z)$

(v) $4x\ (x + 2) - 3x\ (x - 3)$ (vi) $-6a\ (a - 3) - 3\ (a - 1 + b)$

(2) Simplify.

(i) $-(y + 1) - 3\ (y + 2)$ (ii) $-3\ (y - 2) - 3\ (6 - y)$

(iii) $-(2 - a) - 3\ (a + 8)$ (iv) $-x\ (x + 3) - 2x\ (1 - x)$

(v) $a\ (a + 6) - a\ (a + 2)$ (vi) $a\ (2a - 1) - a\ (6 - a)$

5.8 Substituting given values for each unknown in an algebraic expression with up to three unknowns

In Grade 7 you learnt that replacing an unknown term of an algebraic expression by a numerical value is called substitution. By substitution, an algebraic expression takes a numerical value.

Now let us substitute numerical values for the unknown terms of an algebraic expression with three unknowns and find its value.

Let us find the value of the algebraic expression $2p + q - r + 1$ when $p = 4$, $q = 2$ and $r = -3$.

$$\begin{aligned} 2p + q - r + 1 &= 2 \times 4 + 2 - (-3) + 1 \\ &= 8 + 2 + 3 + 1 \\ &= 14 \end{aligned}$$

Now let us find the value of an algebraic expression with brackets by substituting numerical values for the unknowns.

Let us find the value of $3(x + y) + z$ when $x = 2$, $y = 5$ and $z = 10$,

$$\begin{aligned} 3(x + y) + z &= 3(2 + 5) + 10 \\ &= 3 \times 7 + 10 \\ &= 21 + 10 \\ &= 31 \end{aligned}$$

or

$$\begin{aligned} 3(x + y) + z &= 3x + 3y + z \\ &= 3 \times 2 + 3 \times 5 + 10 \\ &= 6 + 15 + 10 \\ &= 31 \end{aligned}$$

Example 1

Find the value of the algebraic expression $2x - y - 2z$ when $x = 4$, $y = 3$ and $z = 2$.

$$\begin{aligned} 2x - y - 2z &= 2 \times 4 - 1 \times 3 - 2 \times 2 \\ &= 8 - 3 - 4 \\ &= 1 \end{aligned}$$

Example 2

Find the value of the algebraic expression $-p + 2q - 3r + 7$ when $p = 5$, $q = -2$ and $r = -3$.

$$\begin{aligned} -p + 2q - 3r + 7 &= -1 \times 5 + 2 \times (-2) - 3 \times (-3) + 7 \\ &= (-5) + (-4) - (-9) + 7 \\ &= (-9) + (+9) + 7 \\ &= 0 + 7 \\ &= 7 \end{aligned}$$

Example 3

Find the value of the algebraic expression $6\,(2a - b) - c$ when $a = 4$, $b = 5$ and $c = 8$.

$$\begin{aligned} 6\,(2a - b) - c &= 6\,(2 \times 4 - 5) - 8 \\ &= 6\,(8 - 5) - 8 \\ &= 6 \times 3 - 8 \\ &= 18 - 8 = 10 \end{aligned}$$

Example 4

Find the value of the algebraic expression $10\,(k - l) + r$ when $k = 4$, $l = 1$ and $r = -3$.

$$\begin{aligned} 10\,(k - l) + r &= 10\,(4 - 1) + (-3) \\ &= 10 \times 3 - 3 \\ &= 30 - 3 = 27 \end{aligned}$$

Example 5

Simplify the algebraic expression $5x + 3y - 4x - y + 8$ and find its value when $x = 2$ and $y = -1$.

$$\begin{aligned} 5x + 3y - 4x - y + 8 &= 5x - 4x + 3y - y + 8 \\ &= x + 2y + 8 \end{aligned}$$

Substituting the given values,

$$\begin{aligned} x + 2y + 8 &= 2 + 2\,(-1) + 8 \\ &= 2 + (-2) + 8 \\ &= 0 + 8 = 8 \end{aligned}$$

Example 6

Simplify the algebraic expression $4\,(a - 2b) + 2\,(b - 3c)$ and find its value when $a = 3$, $b = 1$, $c = -1$.

Expanding the expression,

$$\begin{aligned} 4\,(a - 2b) + 2\,(b - 3c) &= 4 \times a - 4 \times 2b + 2 \times b - 2 \times 3c \\ &= 4a - 8b + 2b - 6c \\ &= 4a - 6b - 6c \end{aligned}$$

Substituting the given values,

$$\begin{aligned} 4a - 6b - 6c &= 4 \times 3 - 6 \times 1 - 6 \times (-1) \\ &= 12 - 6 + 6 \\ &= 12 \end{aligned}$$

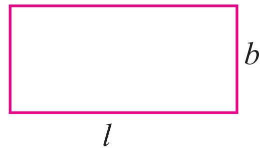
Exercise 5.8

(1) Find the value of each algebraic expression when $x = -3,\ y = -1,\ z = 0$

(i) $x + y$ (ii) $y + 3z + 7$ (iii) $x - 4y + 4z$

(iv) $x + y - z$ (v) $z\ (2x - 3y)$ (vi) $5y - 4z + 3x$

(2) In the given rectangle, the length is l cm and the breadth is b cm.



(i) Write an algebraic expression for its perimeter.

(ii) Find the perimeter of the rectangle if $l = 10$ cm and $b = 7$ cm.

(iii) Find the perimeter of the rectangle if $b = 5$ cm and l is twice b.

(iv) Find the perimeter of the rectangle if $b = 12$ cm and l is 8 cm more than b.

(3) $2x - 9y - 4z + 7$

(i) Find the value of the above algebraic expression when $x = 4,\ y = 3$ and $z = -2$.

(ii) Find the value of the above algebraic expression when $x = 10,\ y = 15$ and $z = -1$.

(iii) Find the value of the above algebraic expression when $x = -4,\ y = -3$ and $z = -2$.

(iv) Find the value of the above algebraic expression when $x = 2,\ y = -3$ and $z = 0$.

(4) Complete the tables given below.

(a)

Expression	Values of the unknowns	Value of the algebraic expression
$3x + 2y + 10$	$x = 4,\ y = 3$	
$2p - 3q - 4r$	$p = 1,\ q = 2,\ r = -3$	
$4a - b + 5c$	$a = 2,\ b = -4,\ c = 1$	

(b)

Expression	Values of the unknowns	Value of the algebraic expression
$3(x + y) + 10z$	$x = -1,\ y = 3,\ z = 2$	
$4(a + 3b) + c$	$a = 5,\ b = 1,\ c = -10$	
$10(m + n) - k$	$m = 3,\ n = -1,\ k = 8$	
$100 - 3(p + 2q)$	$p = 4,\ q = -5$	
$2(a + 2b) + 5(a - b)$	$a = 4,\ b = -1$	

(5) Expand each algebraic expression given below and find its value by substituting the given values for the unknowns.

(i) Find the value of $10(a + 2b) + 3(a - 5b)$ when $a = 7$ and $b = 1$.

(ii) Find the value of $4(m + 3n) + m + 5n$ when $m = 9$ and $n = -2$.

(iii) Find the value of $7(2p - q) - 10p + 3q - 8$ when $p = 2$ and $q = 3$.

(iv) Find the value of $3(2a + 7b) + 3(b + 3c) - 10$ when $a = 1$, $b = 2$ and $c = -3$.

(v) Find the value of $4(x - 5y) - 3(7 - x) + 8l$ when $x = 8$, $y = -1$ and $l = -2$.

Summary

- When multiplying an algebraic expression by a number, each term of the algebraic expression needs to be multiplied by the number. That is, the coefficient of each algebraic term should be multiplied by the given number and simplified.
- When multiplying an algebraic term by an algebraic term, their coefficients are multiplied first and then the unknowns are multiplied.
- When multiplying an algebraic expression by an algebraic term, every algebraic term of the algebraic expression needs to be multiplied by the given algebraic term.
- By substituting values for the unknown terms of an algebraic expression, we obtain a numerical value for the algebraic expression.

6

Solids

By studying this lesson you will be able to,

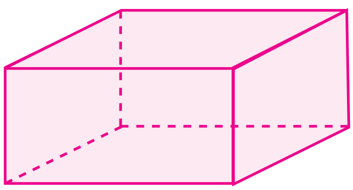
- prepare models of regular octahedrons, regular dodecahedrons and regular icosahedrons,
- verify Euler's relationship for the above solids by considering the number of edges, vertices and faces of these solids, and
- from given, solids identify the platonic solids and describe their characteristics.

6.1 Solids

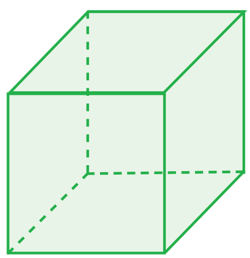
You have learnt that an object which has a specific shape and which occupies a certain amount of space is called a solid object.

You have also learnt that the surfaces of solids objects are plane surfaces or curved surfaces.

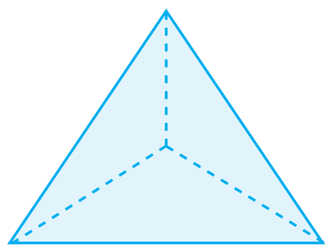
Some solids you have studied in Grades 6 and 7 are illustrated below.



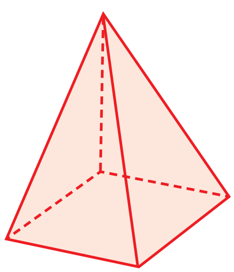
A cuboid



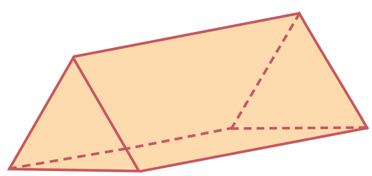
A cube



A regular tetrahedron



A square pyramid



A triangular prism

Do the review exercise to recall what you have learnt about solids in Grades 6 and 7.

Review Exercise

(1) Fill in the blanks in the table given below.

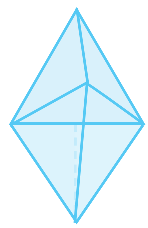
Solid	Number of edges	Number of faces	Number of vertices
Cuboid	12	6	8
Cube			
Regular tetrahedron			
Square pyramid			
Triangular prism			

(2) Draw nets that can be used to construct the following solids.

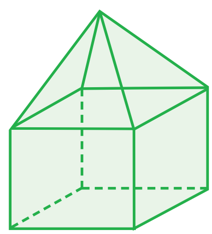
(i) Square pyramid

(ii) Triangular prism

(3) A figure of a solid constructed by pasting together two triangular faces of two identical regular tetrahedrons, one on the other, is given here. Find the number of edges, faces and vertices of this solid.

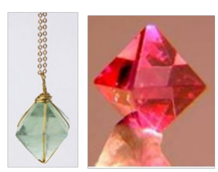


(4) A solid constructed by joining a cube and a square pyramid is shown in the figure. Find the number of edges, faces and vertices of this solid.



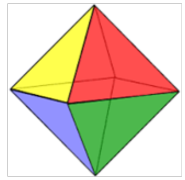
6.2 Octahedron

Diamonds and certain other gems used in jewellery are cut in the shape of an octahedron.



A solid which has 8 faces is called an **octahedron**.

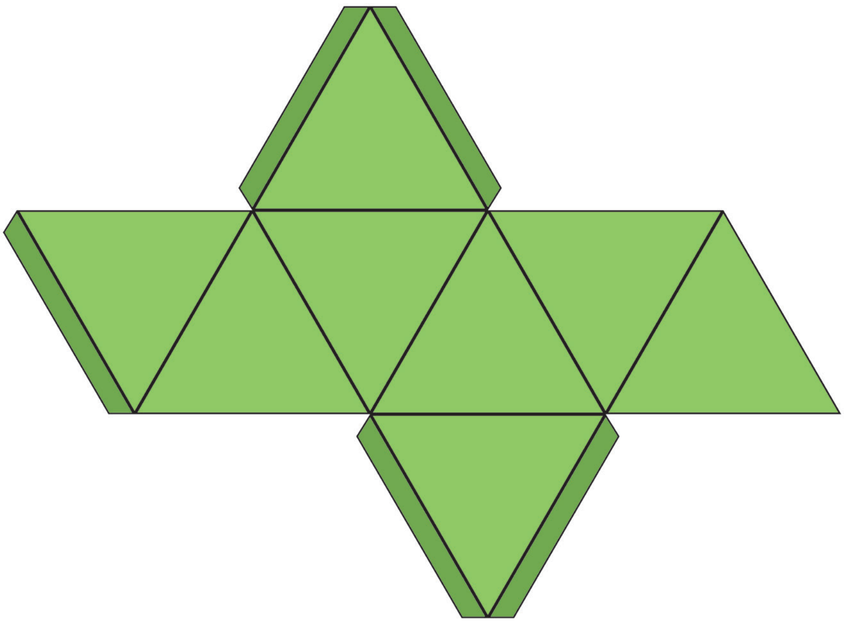
A solid object which has eight identical equilateral triangular shaped faces is called a **regular octahedron**. The figure shows a regular octahedron.



Let us identify the characteristics of a regular octahedron by engaging in the following activity.

Activity 1

Step 1 - Copy the given figure on a thick piece of paper such as a Bristol board, or get a photo copy of the figure and paste it on a thick piece of paper.

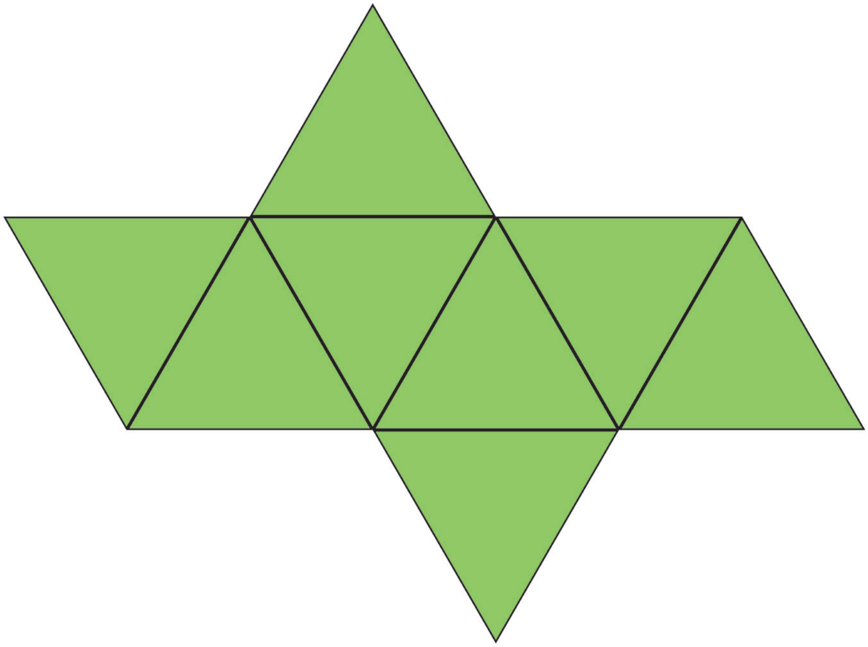


Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a regular octahedron by folding along the edges and pasting along the pasting allowances.

Step 3 - By considering the model you prepared, find the number of faces, edges and vertices of a regular octahedron. Examine and identify the special features of the model.

Step 4 - Write the special features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure, which was used to prepare a model of a regular octahedron, is called a **net of the regular octahedron.**



The object you constructed during the above activity is a model of a regular octahedron.

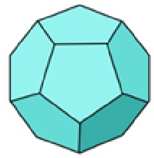
Features you can identify in a regular octahedron

- There are 8 faces in a regular octahedron.
- All faces are the shape of identical equilateral triangles.
- There are 6 vertices in a regular octahedron.
- There are 12 edges in a regular octahedron. All are straight edges.

6.3 Dodecahedron

Models of this shape are used for decorations and ornaments.

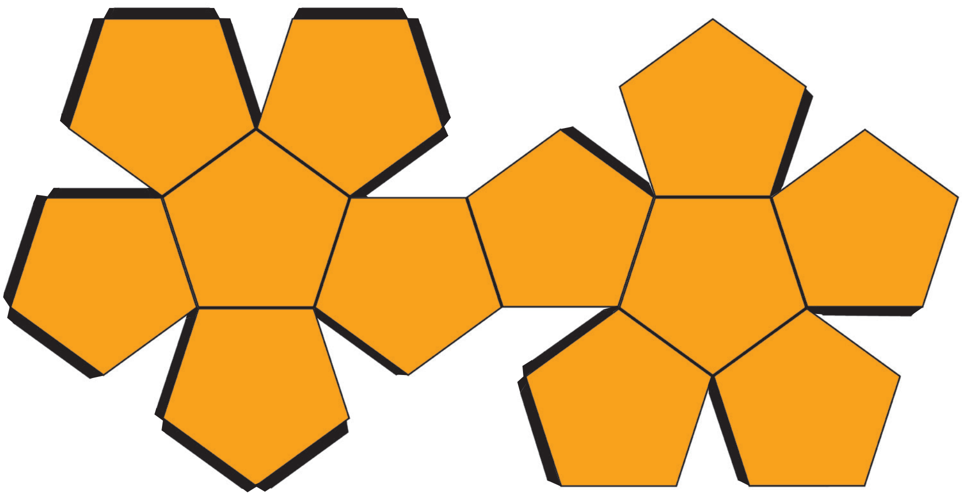
A solid object which has 12 regular pentagonal faces is called a **regular dodecahedron**. The figure shows a regular dodecahedron.



Let us identify the characteristics of a regular dodecahedron by engaging in the following activity.

Activity 2

Step 1 - Copy the given figure on a thick piece of paper such as a Bristol board or get a photo copy of the figure and paste it on a thick piece of paper.

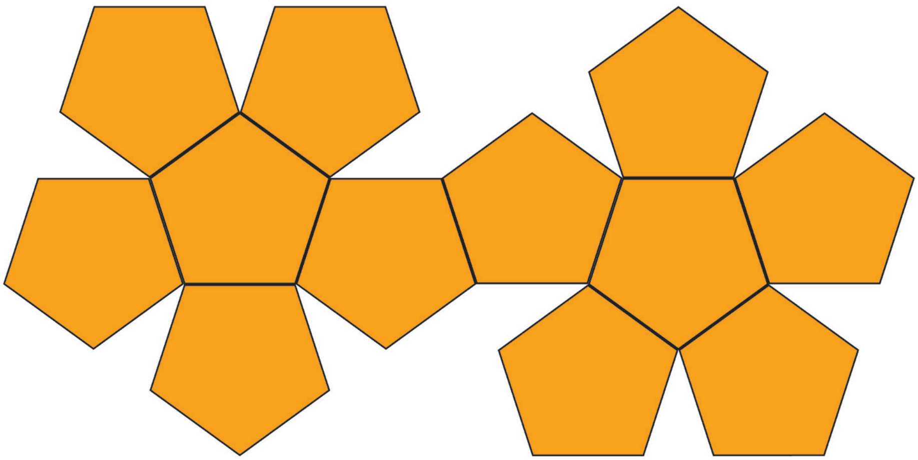


Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a regular dodecahedron by folding along the edges and pasting along the pasting allowances.

Step 3 - By considering the model you prepared, find the number of faces, edges and vertices of a regular dodecahedron. Examine and identify the special features of the model.

Step 4 - Write the special features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a regular dodecahedron, is called a **net of the regular dodecahedron**.



The object you constructed during the above activity is a model of a regular dodecahedron.

Features you can identify in a regular dodecahedron

- There are 12 faces in a regular dodecahedron.
- All the faces of a regular dodecahedron take the shape of identical regular pentagons.
- There are 20 vertices in a regular dodecahedron.
- There are 30 edges in a regular dodecahedron. All are straight edges.

6.4 Icosahedron

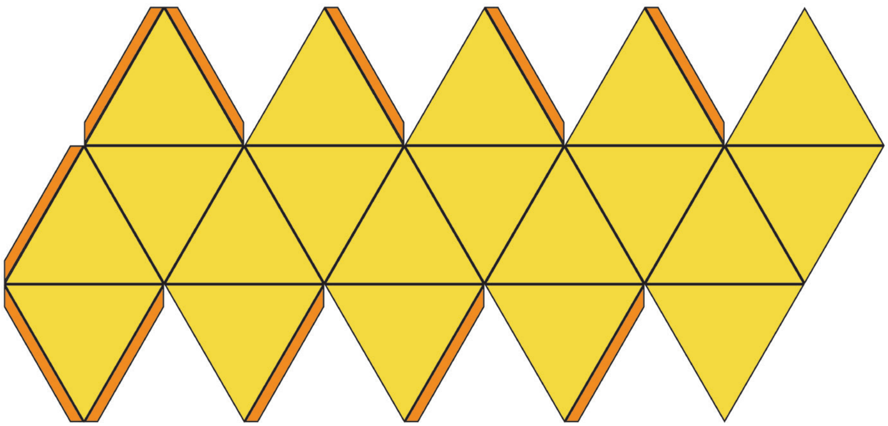
A model which can be used in decorations such as Vesak lanterns is given here. It is known as an icosahedron.

A solid which has twenty equilateral triangular faces is called a **regular icosahedron**. The figure shows a regular icosahedron.

Let us identify the characteristics of a regular icosahedron by engaging in activity 3.

Activity 3

Step 1 - Copy the given figure on a thick piece of paper such as a Bristol board, or get a photo copy of the figure and paste it on a thick piece of paper.

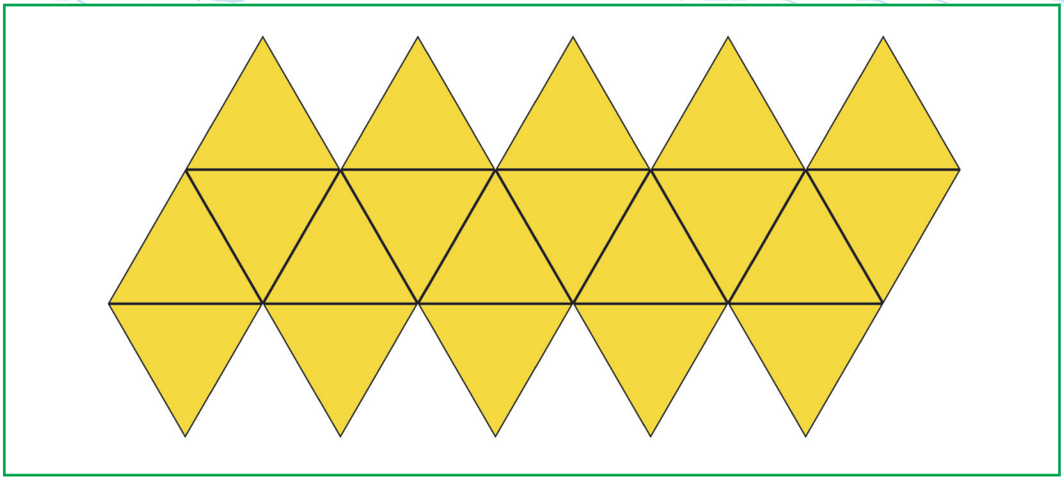


Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a regular icosahedron by folding along the edges and pasting along the pasting allowances.

Step 3 - By considering the model you prepared, find the number of faces, edges and vertices of a regular icosahedron. Examine and identify the special features of the model.

Step 4 - Write the special features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a regular icosahedron is called a **net of the regular icosahedron**.



The object you constructed during the above activity is a model of a regular icosahedron.

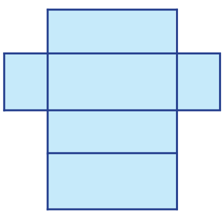
Features you can identify in a regular icosahedron

- There are 20 faces in a regular icosahedron.
- The faces of a regular icosahedron take the shape of identical equilateral triangles.
- There are 12 vertices in a regular icosahedron.
- There are 30 edges in a regular icosahedron. All are straight edges.

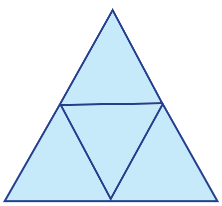
Exercise 6.1

(1) Name the solid which can be constructed using each net given below.

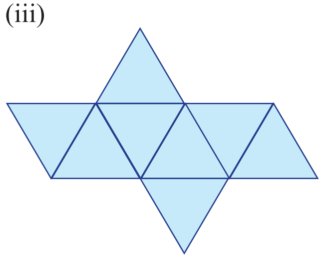
(i)



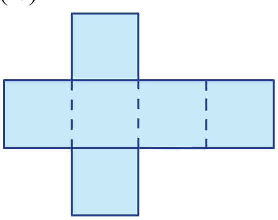
(ii)



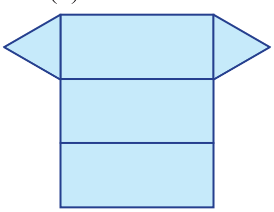
(iii)



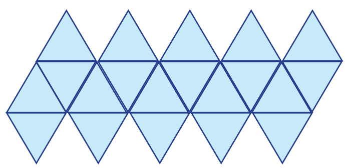
(iv)



(v)



(vi)



6.5 Verification of Euler's relationship for solids

You learnt in Grade 7 about the relationship between the edges, vertices and faces of a solid, which was first presented by the Swiss mathematician Euler. Let us recall what you learnt.

Euler's relationship
In a solid with straight edges, the sum of the number of faces and the number of vertices is two more than the number of edges.

This relationship can be expressed as follows.

Number of Vertices + Number of Faces = Number of Edges + 2

$V + F = E + 2$

Activity 4

Fill in the blanks in the table given below by observing the solids you constructed in activities 1, 2 and 3.

Solid	Number of vertices (V)	Number of faces (F)	Number of edges (E)	$V + F - E$	Is Euler's relationship satisfied?
Regular Octahedron					
Regular Dodecahedron					
Regular Icosahedron					

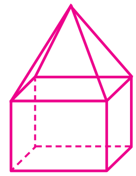
Exercise 6.2

(1) Verify Euler's relationship for a regular tetrahedron by considering the number of faces, vertices and edges it has.

(2) For a square pyramid,
(i) write down the number of faces, vertices and edges.
(ii) show that the above values satisfy Euler's relationship.

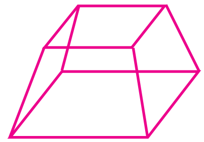
(3) If a certain solid has 9 edges and 6 vertices, and if Euler's relationship is satisfied, find the number of faces it has.

(4) A figure of a composite solid is shown here. Determine with reasons whether Euler's relationship is satisfied for this solid.



(5) A certain solid has 10 edges and 6 faces. Find the number of vertices it has, if Euler's relationship is satisfied.

(6) The figure given here shows a pyramid of which the upper portion has been cut out and removed. Verify Euler's relationship for this solid.

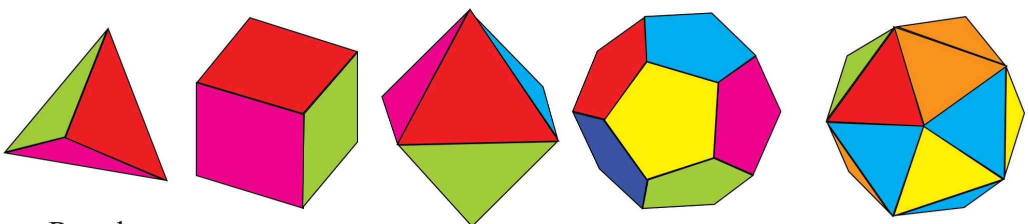


6.6 Platonic solids

Platonic solids are solids having identical regular polygonal faces and with the same number of faces meeting at every vertex.

You have learnt about the five types of solids which are considered as platonic solids. They are the regular tetrahedron, cube, regular octahedron, regular dodecahedron and the regular icosahedron.

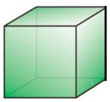
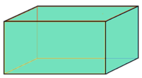
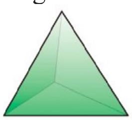
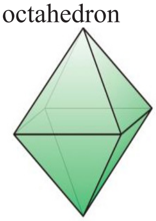
They are called **platonic solids**.

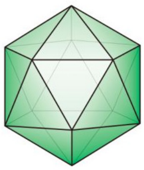
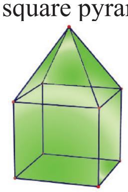


Regular Tetrahedron | Cube | Regular Octahedron | Regular Dodecahedron | Regular Icosahedron

Exercise 6.3

(1) Complete the table given below.

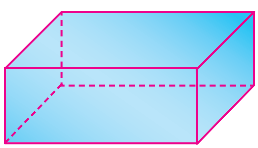
Solid	Shapes of the faces of the solid	Are all the faces regular?	Are the number of faces meeting at each vertex equal?	Number of faces meeting at a vertex	Is the solid a platonic solid?
Cube 	Square	Regular	Equal	3	Yes
Cuboid 					
Regular tetrahedron 					
Regular octahedron 					
Regular dodecahedron 					

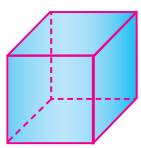
Solid	Shapes of the faces of the solid	Are all the faces regular?	Are the number of faces meeting at each vertex equal?	Number of faces meeting at a vertex	Is the solid a platonic solid?
Regular icosahedron 					
Composite solid consisting of a cuboid and a square pyramid 					

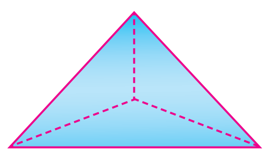
(2) Construct a regular icosahedron and 20 regular tetrahedrons such that the icosahedron and the tetrahedrons have edges of equal length. Construct a composite solid by pasting a tetrahedron on each face of the icosahedron. For the composite figure, find

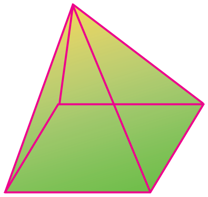
(i) the number of edges.
(ii) the number of faces.
(iii) the number of vertices.

(3) From the following, select the platonic solids and write down the corresponding numbers.

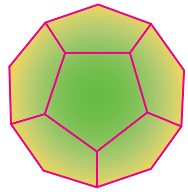

(i)


(ii)

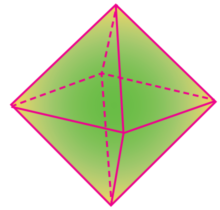

(iii)



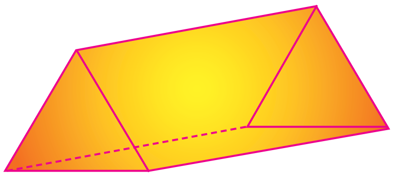
(iv)



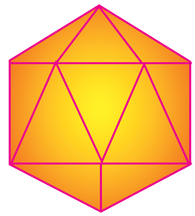
(v)



(vi)



(vii)



(viii)

Summary

- The sum of the number of faces and the number of vertices of a solid with straight edges is 2 more than the number of edges.
- Solids having identical regular polygonal faces and with the same number of faces meeting at every vertex are called platonic solids.
- The five types, regular tetrahedrons, cubes, regular octahedrons, regular dodecahedrons and regular icosahedrons are the only solids that are platonic solids.

Solid	Shape of a face	Number of faces	Number of edges	Number of vertices
Cube	Square	6	12	8
Cuboid	Rectangle	6	12	8
Regular tetrahedron	Equilateral triangle	4	6	4
Square pyramid	One face is square shaped. The other 4 faces take the shape of identical triangles	5	8	5
Triangular prism	3 rectangular faces and 2 triangular faces	5	9	6
Regular octahedron	Equilateral triangle	8	12	6
Regular dodecahedron	Regular pentagon	12	30	20
Regular icosahedron	Equilateral triangle	20	30	12

7 Factors

By studying this lesson you will be able to,

- find the highest common factor of the terms of a set which consists of up to three algebraic terms,
- write an algebraic expression as a product of two factors, where one factor is the highest common factor of the terms of the algebraic expression, and
- establish that an algebraic expression written in terms of its factors is the given algebraic expression, by multiplying the factors.

7.1 The highest common factor (HCF) of several numbers

$6 = 2 \times 3$

You have learnt previously that 2 and 3 are factors of 6.

When a number is written as a product of two whole numbers, those numbers are called **factors** of the original number.

The HCF of two or more numbers is the largest of all the common factors of the given numbers. That is, the largest number by which all the given numbers are divisible is their HCF.

Now let us find the HCF of 6 and 10.

$6 = 1 \times 6$ $\qquad$ $10 = 1 \times 10$

$6 = 2 \times 3$ $\qquad$ $10 = 2 \times 5$

$\therefore$ the factors of 6 are 1, 2, 3 and 6.
The factors of 10 are 1, 2, 5 and 10.

1 and 2 are the common factors of 6 and 10. Since 2 is the larger factor, the HCF of 6 and 10 is 2.

You learnt in Grade 7 how to find the HCF of several numbers by writing each as a product of prime numbers. Let us recall what you learnt through an example.

Let us find the HCF of 6, 12 and 18.

Let us write each number as a product of prime factors.

$$\begin{array}{r|l} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{aligned} 6 &= 2 \times 3 \\ 12 &= 2 \times 2 \times 3 \\ 18 &= 2 \times 3 \times 3 \end{aligned}$$

We obtain the HCF of 6, 12 and 18 by taking the product of the prime factors which are common to these three numbers.

The HCF of 6, 12 and 18 $= 2 \times 3 = 6$

Note

To find the prime factors of a whole number,

- it is sequentially divided by the prime numbers by which it is divisible, starting from the smallest such prime number, till the answer 1 is obtained.

Review Exercise

Find the HCF of each set of numbers given below.

(i) 12, 18 (ii) 30, 24 (iii) 45, 60

(iv) 6, 12, 18 (v) 15, 30, 75 (vi) 36, 24, 60

(vii) 6, 9, 12 (viii) 15, 30, 45 (ix) 11, 13, 5

7.2 The highest common factor of several algebraic terms

Now let us see what is meant by the HCF of several algebraic terms and how to find it.

Let us find the HCF of the algebraic terms $4x$, $8xy$ and $6xyz$.

Let us write each term as a product of its factors.

$$\begin{aligned} 4x &= 2 \times 2 \times x \\ 8xy &= 2 \times 2 \times 2 \times x \times y \\ 6xyz &= 2 \times 3 \times x \times y \times z \end{aligned}$$

Here, the coefficient of each algebraic term is written as a product of its prime factors and the unknowns are separated and written as a product.

The common factors of all three algebraic terms, $4x$, $8xy$ and $6xyz$ are 2 and x.

The HCF of the algebraic terms, $4x$, $8xy$ and $6xyz$ is the product of the factors which are common to all three terms.

$\therefore$ The HCF of $4x$, $8xy$ and $6xyz$ $= 2 \times x$

$= 2x$

Example 1

Find the HCF of the algebraic terms in each part given below.

(i) $2pq,\ 4pqr$ (ii) $7mn,\ 14mnp,\ 28mnq$

(i) $2pq = 2 \times p \times q$

$4pqr = 2 \times 2 \times p \times q \times r$

The HCF of $2pq$ and $4pqr$ $= 2 \times p \times q$

$= 2pq$

(ii) $7mn = 7 \times m \times n$

$14mnp = 2 \times 7 \times m \times n \times p$

$28mnq = 2 \times 2 \times 7 \times m \times n \times q$

The HCF of $7mn$, $14mnp$ and $28mnq$ $= 7 \times m \times n$

$= 7mn$

Exercise 7.1

Find the HCF of the algebraic terms in each part given below.

(i) $xy,\ 3xy,\ 4x$ (ii) $4c,\ 8a,\ 4b$
(iii) $2x,\ 8x,\ 4xy$ (iv) $4p,\ 8pq,\ 12pq$
(v) $8pqr,\ 16qr,\ 7mqr$ (vi) $4x,\ 6xy,\ 8qrx$
(vii) $4x,\ 6abx,\ 10abxy$ (viii) $6mn,\ 12mny,\ 15my$

7.3 Writing an algebraic expression as a product of its factors

Since 2 and 3 are the prime factors of 6, 6 can be written as a product of its prime factors as $6 = 2 \times 3$.

Now let us consider how to write an algebraic expression as a product of its factors.

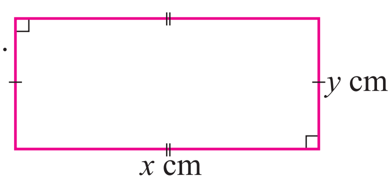
Let us find the perimeter of the rectangle in the figure.

Method I

Let us add the lengths of all four sides of the rectangle.

Perimeter of the rectangle $= x + y + x + y$

$= 2x + 2y$



Method II

Let us find the perimeter by multiplying the sum of the length and breadth of the rectangle by two.

Perimeter of the rectangle $= (x + y) \times 2$
$= 2(x + y)$

Since the perimeter of the same rectangle is found by both methods, the two expressions obtained for the perimeter are equal.

$$\therefore\ 2x + 2y = 2(x + y)$$

Writing the algebraic expression $2x + 2y$ as $2(x + y)$, is called **writing the algebraic expression $2x + 2y$ as a product of factors**.

That is, 2 and $(x + y)$ are two factors of the expression $2x + 2y$.

➢ Now let us write the algebraic expression $12x + 18y$ as a product of two factors.

$12x + 18y$ can be expressed as a product of two factors in several ways.

(i) $12x + 18y = 2 \times 6x + 2 \times 9y$
$= 2(6x + 9y)$

In this instance, 2 is taken as a common factor of the two terms.

(ii) $12x + 18y = 3 \times 4x + 3 \times 6y$
$= 3(4x + 6y)$

In this instance, 3 is taken as a common factor of the two terms.

(iii) $12x + 18y = 6 \times 2x + 6 \times 3y$
$= 6(2x + 3y)$

In this instance, 6 is taken as a common factor of the two terms.

Since there is no common factor in $2x$ and $3y$, which are the terms of the expression within brackets, 6 is the HCF of the terms $12x$ and $18y$.

When writing such an algebraic expression as a product of factors, the convention is to write the first factor as a number which is the HCF of the terms of the given expression, and the other factor as an algebraic expression, where the HCF of its terms is 1.

Accordingly, when writing an algebraic expression as a product of factors,

- first find the highest common factor of the terms of the algebraic expression,
- take this HCF as one factor and the expression which is obtained by dividing each term of the algebraic expression by this HCF as the other factor, and
- write the algebraic expression as a product of these two factors.

Example 1

Write the expression $36a + 60b$ as a product of factors.

$$36a = 2 \times 2 \times 3 \times 3 \times a$$
$$60b = 2 \times 2 \times 3 \times 5 \times b$$

The HCF of the terms $36a$ and $60b$ $= 2 \times 2 \times 3$
$= 12$

$\therefore\ 36a + 60b = 12 \times 3a + 12 \times 5b$
$= 12\,(3a + 5b)$

$36a \div 12 = 3a$
$60b \div 12 = 5b$

Example 2

Write the expression $12x + 20y + 16z$ as a product of factors.

$$12x = 2 \times 2 \times 3 \times x$$
$$20y = 2 \times 2 \times 5 \times y$$
$$16z = 2 \times 2 \times 2 \times 2 \times z$$

The HCF of $12x$, $20y$ and $16z$ $= 2 \times 2$
$= 4$

$\therefore\ 12x + 20y + 16z = 4 \times 3x + 4 \times 5y + 4 \times 4z$
$= 4\,(3x + 5y + 4z)$

$12x \div 4 = 3x$
$20y \div 4 = 5y$
$16z \div 4 = 4z$

Exercise 7.2

(1) Fill in the blanks.

(i) $3x + 12 = 3 \times \square + 3 \times \square = 3\,(\square + \square)$
(ii) $15x + 20y = 5 \times \square + 5 \times \square = 5\,(\square + \square)$
(iii) $12a + \square = 6 \times \square + 6 \times \square = 6\,(\square + 3)$
(iv) $12x + 8y + 20z = 4 \times \square + 4 \times \square + 4 \times \square = 4\,(\square + \square + \square)$
(v) $30x + 24y + 18 = \square\,(5x + \square + \square)$

(2) Write each of the algebraic expressions given below as a product of two factors such that one factor is the HCF of the terms of the expression.

(a) (i) $2x + 6y$ (ii) $8x + 12y$ (iii) $15a + 18b$
(iv) $9x + 27y$ (v) $4p + 20q$ (vi) $12p + 30q$
(vii) $20a - 30b$ (viii) $36a - 54b$ (ix) $60p - 90q$

(b) (i) $5x - 10y + 25$ (ii) $3a + 15b - 12$ (iii) $18 - 12m + 6n$
(iv) $10a - 20b - 15$ (v) $9c - 18a + 9$ (vi) $12d + 6 + 18c$
(vii) $3x + 6y - 3$ (viii) $10m + 4n - 2$ (ix) $12a - 8b + 4$
(x) $9 + 3b + 6c$ (xi) $3a^2 - 6ab + 9b^2$ (xii) $4a^2 - 16ab - 12c$

7.4 Writing an algebraic expression as a product of factors where one factor is a negative number

Since $(-12) = (-6) \times 2$, we have that (-6) is a factor of (-12).

Since $(-12) = 6 \times (-2)$, (-2) is also a factor of (-12).

Since $12 = (-6) \times (-2)$, both (-6) and (-2) are factors of 12.

Example 1

(i) Write (-15) as a product of two factors, such that (-3) is a factor.

$(-15) = (-3) \times 5$

(ii) Write 10 as a product of two factors such that (-2) is a factor.

$10 = (-2) \times (-5)$

Accordingly, (-2) and (-5) are two factors of 10.

Now let us consider an instance where one factor of the algebraic expression is a negative number.

Let us consider the algebraic expression $-2x + 6y$. Here, 2 is a common factor of $-2x$ and $6y$.

Therefore, $-2x + 6y = 2(-x + 3y)$

Since $-2x = (-2) \times x$ and $6y = (-2) \times (-3) \times y$,

(-2) is also a common factor of $-2x$ and $6y$.

$$\begin{aligned} \therefore -2x + 6y &= (-2) \times x + (-2) \times (-3)\, y \\ &= (-2)\,(x + (-3)\, y) \\ &= -2\,(x - 3y) \end{aligned}$$

$\therefore$ the algebraic expression $-2x + 6y$ can also be written as a product of two factors as $-2(x - 3y)$.

Example 2

Write down each of the algebraic expressions given below as a product of two factors such that one factor is a negative number.

(i) $-4x - 16y$ (ii) $-8m + 24n - 16$

$$\begin{aligned} \text{(i) } -4x - 16y &= (-4) \times x + (-16)y \\ &= (-4) \times x + (-4) \times (+4)\, y \\ &= (-4)\,(x + (+4)\, y) \\ &= -4\,(x + 4y) \end{aligned}$$

$$\begin{aligned} \text{(ii) } -8m + 24n - 16 &= -8 \times 1m + (-8) \times (-3)\, n + (-8) \times (+2) \\ &= -8\,(m - 3n + 2) \end{aligned}$$

Note

When one factor is a negative number, the sign of each term of the other factor is opposite to that of the corresponding term in the original algebraic expression.

Exercise 7.3

(1) (i) Write (–20) as a product of two factors such that (–4) is one of the factors.

(ii) Write 12 as a product of two factors such that (–4) is one of the factors.

(2) Write each algebraic expression given below as a product of two factors such that one factor is a negative number.

(i) $12x - 4y$ (ii) $-12x + 4y$ (iii) $-12x - 4y$

(iv) $-3a + 15b - 6c$ (v) $-12a + 18b - 24c$ (vi) $-8p + 40q - 24$

7.5 More on writing an algebraic expression as a product of two factors

Let us consider the algebraic expression $pq + pr$.

$pq = p \times q$

$pr = p \times r$

Since p is a factor of each term of this expression, p is a common factor of the two terms.

$\therefore pq + pr = p \times q + p \times r$

$= p(q + r)$

Accordingly, when writing an algebraic expression as a product of factors,

- first find the HCF of the terms of the algebraic expression,
- take the HCF as one factor and the expression which is obtained by dividing each term of the algebraic expression by the HCF as the other factor, and
- write the algebraic expression as a product of these two factors.

Example 1

Write the expression $18x + 24xy + 12xz$ as a product of two factors.

The HCF of the terms $18x$, $24xy$ and $12xz$ is $6x$

$\therefore 18x + 24xy + 12xz = 6x \times 3 + 6x \times 4y + 6x \times 2z$

$= 6x(3 + 4y + 2z)$

Note

- Let us simplify 6 ÷ 9.

You have learnt that $6 \div 9 = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$.

Moreover, this can also be simplified as $\frac{6}{9} = \frac{{}^{1}\cancel{3} \times 2}{{}_{1}\cancel{3} \times 3} = \frac{2}{3}$.

- Let us simplify $3xy \div 5y$.

$3xy \div 5y = \frac{3xy}{5y} = \frac{3 \times x \times y}{5 \times y}$

Since y represents a number, it can be simplified as above.

$\frac{3 \times x \times \cancel{y}^{1}}{5 \times \cancel{y}_{1}} = \frac{3 \times x}{5} = \frac{3x}{5}$

Example 2

Write the expression $15pq + 45qr + 60q$ as a product of factors.

$15pq = 3 \times 5 \times p \times q$
$45qr = 3 \times 3 \times 5 \times q \times r$
$60q = 2 \times 2 \times 3 \times 5 \times q$

The HCF of $15pq$, $45qr$ and $60q$ $= 3 \times 5 \times q$
$= 15q$

$\therefore\ 15pq + 45qr + 60q = 15q\,(p + 3r + 4)$

$15pq \div 15q = p$
$45qr \div 15q = 3r$
$60q \div 15q = 4$

Example 3

Write the expression $3a + 6ab + 12ac$ as a product of factors.

Here $3a = 3 \times a$
$6ab = 3 \times 2 \times a \times b$
$12ac = 2 \times 2 \times 3 \times a \times c$

HCF of $3a$, $6ab$ and $12ac$ $= 3 \times a$

When the HCF $3a$ is separated out as a common factor and written we obtain,

$3a + 6ab + 12ac = 3a\,(1 + 2b + 4c)$.

Note that when the expression within brackets is multiplied by $3a$, the original expression, $3a + 6ab + 12ac$ is obtained.

$3a\,(1 + 2b + 4c) = 3a + 6ab + 12ac$

$\therefore$ $3a + 6ab + 12ac$ is the product of the two factors $3a$ and $(1 + 2b + 4c)$.

Exercise 7.4

(1) Write each algebraic expression given below as a product of two factors.

(i) $ab + ac$ (ii) $p + pq$ (iii) $xyz + xpq$

(iv) $3x + 6xy$ (v) $15pq - 20pr$ (vi) $4p - 16pq + 12pr$

(vii) $2a - 8ab - 8ac$ (viii) $5x - 10xy - 5xz$ (ix) $3ab - 9abc$

(2) Write each of the following algebraic expressions as a product of two factors Establish the accuracy of your answer by simplifying the product.

(i) $xyz - 2xyp$ (ii) $12x - 20xy$ (iii) $ab + ac - ad$

(iv) $p + pq + pqr$ (v) $xp - xy - x$ (vi) $6ab - 8ab^2 + 12ac$

(3) Join each algebraic expression in group A with the algebraic expression in group B which it is equal to.

A	B
(i) $2\,(x + 2y + 5)$	$10a - 2ac + 4ab$
(ii) $4\,(2a + b + 3c)$	$15xyz - 25xy + 20xz$
(iii) $5\,(2a - 1 + 3b)$	$4p^2r + 2qr + 2pqr$
(iv) $4\,(3x - 2y + 5z)$	$12x - 8y + 20z$
(v) $4p\,(a + b + 1)$	$2x + 4y + 10$
(vi) $2a\,(5 - c + 2b)$	$12x - 6xy + 9xz$
(vii) $x\,(2 - 3y + 3y^2)$	$8a + 4ab - 4ac$
(viii) $4a\,(2 + b - c)$	$4ap + 4bp + 4p$
(ix) $5x\,(3yz - 5y + 4z)$	$10a - 5 + 15b$
(x) $3x\,(4 - 2y + 3z)$	$8a + 4b + 12c$
(xi) $2r\,(2p^2 + q + pq)$	$2x - 3xy + 3xy^2$

(4) Complete the table given below.

Original expression	After factoring the expression
....	$4\,(3a + 2b + 3a^2)$
$9a + 27ac^2 + 18ab$	....
....	$3a\,(2p + 3r + 6)$
....	$2a\,(a + 3b + 2ac)$
$8xy + 24xp + 40xq$	....
....	$2\,(3ab + 4bc - 5ac)$
....	$3x\,(2pq + 3x + 6p)$
....	$6\,(2xy^2 + 3xy + 4z)$
$3ab - 6ab + 12ac$	....
$8xy - 12px - 20axy$	....

(5) Fill in the blanks in the table.

Algebraic expression	One factor of the algebraic expression	As a product of two factors
$-4x + 12$	4	....
$-4x + 12$	-4	....
$-6x + 8y$	2	....
$-6x + 8xy$	$-2x$	....
$-2a + 4b - 6c$	2	....
$-2a + 4b - 6c$	-2	....
$-3ab - 9b$	$-3b$	....
$2xy - 8xyz$	$2xy$	....
$5xy + 10xy + 10py$	....	....

Summary

When writing an algebraic expression as a product of factors,

- first find the HCF of the terms of the algebraic expression,
- take the HCF as one factor and the expression which is obtained by dividing each term of the algebraic expression by the HCF as the other factor, and
- write the algebraic expression as a product of these two factors.

8

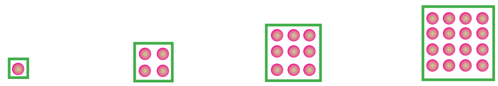
Square Root

By studying this lesson, you will be able to,

- identify what a perfect square is,
- write the square of each of the whole numbers from 1 to 20,
- find the square root of the perfect squares from 1 to 1000 by observation and by considering their prime factors.

8.1 Square of a positive integer

A few numbers which can be represented by a square shaped arrangement of dots are given below.

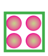
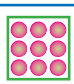
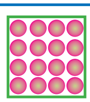


Rows 1 Columns 1   Rows 2 Columns 2   Rows 3 Columns 3   Rows 4 Columns 4

You have learnt earlier that the numbers 1, 4, 9, 16, … which can be represented as above are called square numbers.

We get the square numbers 1, 4, 9, 16, ... by multiplying each positive integer by itself. We can write these square numbers using indices as 1^2, 2^2, 3^2, 4^2, ...

These are read as the "one squared", "two squared" etc.

Representation of the square number	Number of rows/ columns	How the square is obtained	Square of the number, using indices	Square of the number
	Rows 1, Columns 1	1×1	1^2	1
	Rows 2, Columns 2	2×2	2^2	4
	Rows 3, Columns 3	3×3	3^2	9
	Rows 4, Columns 4	4×4	4^2	16

The number we obtain by multiplying a number by itself, is called the **square** of that number. The square of a positive integer is called a **perfect square**.

1, 4, 9, 16, ... are the squares of the numbers 1, 2, 3, 4, ... Therefore they are perfect squares.

Example 1

A square tile is of side length 8 cm. Show that the numerical value of its surface area is a perfect square.

The length of a side of the square tile = 8 cm

Its surface area = 8 cm × 8 cm

= 64 cm^2

The numerical value of the area of the square tile = 64 = 8 × 8

64, is the square of 8, so the numerical value of the surface area of the tile is a perfect square.

Exercise 8.1

(1) Represent the square of 5 by an arrangement of dots and write down its value.

(2) Complete the table given below and answer the questions accordingly.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Square of the number																	

By adding some pairs of perfect squares in the second row, we can obtain another perfect square. Observe the table and write four such relationships.

$9 + 16 = 25$

$\therefore \; 3^2 + 4^2 = 5^2$

.... + ... = ...

∴ .... + ... = ...

.... + ... = ...

.... + ... = ...

(3) (i) Write down the perfect square between 10 and 20, and the reason for it.

(ii) Write down the perfect square between 60 and 70, and the reason for it.

(iii) Write down the perfect square between 80 and 90, and the reason for it.

(iv) How many perfect squares are there between 110 and 160?

(4) Complete the table given below.

Odd numbers added consecutively	Sum	The perfect square in index form
1		
1 + 3	4	2^2
1 + 3 + 5		
1 + 3 + 5 + 7		
1 + 3 + 5 + 7 + 9		

Using the above table, write the special feature of the numbers that are obtained when consecutive odd integers starting from 1 are added together.

8.2 The digit in the units place of a perfect square

The table below shows the squares of the numbers from 1 to 15.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Perfect Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
The digit in the units place of the perfect square	1	4	9	6	5	6	9	4	1	0	1	4	9	6	5

- The digit in the units place of the square of a positive integer is the digit in the units place of the square of the digit at the right end (units place) of that positive integer.
- The digit in the units place of a square number is one of the numbers in the 3rd row of the above table.

- It is clear from the 3rd row of the table, that the digit in the units place of a perfect square is one of the digits 0, 1, 4, 5, 6 or 9.
- None of the digits 2, 3, 7 and 8 is ever the digit in the units place of a perfect square.

Example 1

Is 272 a perfect square?

If the digit in the units place of a whole number is 2, 3, 7 or 8, then that number is not a perfect square. In 272, the digit in the units place is 2. Therefore, it is not a perfect square.

Exercise 8.2

(1) By considering the digit in the units place of each of the numbers given below, show that they are not perfect squares.

(i) 832 (ii) 957 (iii) 513

(2) Give an example of a perfect square which has 9 in its units place.

(3) "If the digit in the units place of a whole number is one of 0, 1, 4, 5, 6 or 9, then it is a perfect square". Show with an example that this statement is not always true.

(4) Observe the digit in the units place of each number given below and write the digit in the units place of their respective squares.

(i) 34 (ii) 68 (iii) 45

8.3 The square root of a perfect square

$16 = 4 \times 4 = 4^2$. Since16 is the square of 4, the square root of 16 is 4.

$49 = 7^2$, so the square root of 49 is 7.

$81 = 9^2$, so the square root of 81 is 9.

To indicate the square root of a number, we use the symbol "$\sqrt{\ }$" .

Accordingly; the square root of 16 $= \sqrt{16} = \sqrt{4^2} = 4$,
the square root of 25 $= \sqrt{25} = \sqrt{5^2} = 5$,
the square root of 100 $= \sqrt{100} = \sqrt{10^2} = 10$,

the square root of 4 $= \sqrt{4} = 2$ (because $2^2 = 4$)
the square root of 1 $= \sqrt{1} = 1$ (because $1^2 = 1$)

If $c = a^2$ where a is a positive number, then $\sqrt{c} = a$. That is, a is the square root of c.

If a number is the square of a positive number, then the second number is the square root of the first.

The square roots of perfect squares such as 36, 49, 64 can be expressed quickly from memory. However it is not easy to do the same for every perfect square.

We have to use different methods to find them.
Let us see how we can find the square root,

- by using prime factors, and
- by observation.

• Finding the square root of a perfect square using prime factors

Let us find the value of $\sqrt{36}$ using prime factors.

Let us first write 36 as a product of its prime factors.

$$36 = 2 \times 2 \times 3 \times 3$$
$$36 = (2 \times 3) \times (2 \times 3)$$
$$= (2 \times 3)^2$$
$$\therefore \sqrt{36} = 2 \times 3$$
$$= 6$$

2	36
2	18
3	9
3	3
	1

Example 1

Find the value of $\sqrt{576}$ using prime factors.

$$576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$= (2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 3)$$
$$= (2 \times 2 \times 2 \times 3)^2 \quad \text{or} \quad 576 = 24^2$$
$$\therefore \sqrt{576} = 2 \times 2 \times 2 \times 3 \quad \text{or} \quad \sqrt{576} = 24$$
$$= 24$$

Exercise 8.3

(1) Find the value of each of the following.

(i) $\sqrt{(2 \times 5)^2}$ (ii) $\sqrt{(2 \times 3 \times 5)^2}$ (iii) $\sqrt{(3 \times 5) \times (3 \times 5)}$

(iv) $\sqrt{3 \times 3 \times 7 \times 7}$ (v) $\sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3}$

(2) Find the square root using prime factors.

(i) 144 (ii) 400 (iii) 900

(iv) 324 (v) 625 (vi) 484

(3) What is the side length of a square shaped parking lot of area 256 m^2?

(4) The area of a square land is 169 m^2. Find the length of a side of the land.

• Finding the square root of a perfect square by observation

➢ Finding the digit in the units place of the square root of a perfect square

Activity 1

(1) Complete the table given below by considering the perfect squares you have identified so far and their square roots.

(i)	Perfect squares with the digit 1 in the units place	1	81	121	361	441
	Square roots of these perfect squares	1	9	11	19	21
(ii)	Perfect squares with the digit 4 in the units place					
	Square roots of these perfect squares					
(iii)	Perfect squares with the digit 5 in the units place					
	Square roots of these perfect squares					
(iv)	Perfect squares with the digit 6 in the units place					
	Square roots of these perfect squares					
(v)	Perfect squares with the digit 9 in the units place					
	Square roots of these perfect squares					
(vi)	Perfect squares with the digit 0 in the units place					
	Square roots of these perfect squares					

(2) Complete the table given below using the information in (i) to (vi) in the above table.

Digit in the units place of the perfect square	Digit in the units place of the square root
1	
4	
5	
6	
9	
0	

According to the above activity, the digit in the units place of the square root of a perfect square, which depends on the digit in the units place of the perfect square, is as follows.

Digit in the units place of the perfect square	Digit in the units place of the square root
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7
0	0

➢ Finding the digit in the tens place of the square root of a perfect square between 101 and 1000

Since $40 \times 40 = 1600$, the square root of a number between 101 and 1000 will be less than 40. Therefore, the square root of such a number will only have digits in the units place and the tens place.

The digit in the tens place of the square root of a perfect square is as follows.

- If the digit in the hundreds place of a number is a perfect square, then the square root of that digit is the digit in the tens place of the answer (the square root of the perfect square).
- If the digit in the hundreds place of a number is not a perfect square, then the square root of the perfect square which is closest and less than the digit in the hundreds place, is the digit in the tens place of the answer.

Example 1

Find $\sqrt{961}$.

- Since the digit in the units place is 1, the digit in the units place of the square root must be 1 or 9.
- The digit in the hundreds place of the given number is 9, so the digit in the tens place of the square root is $\sqrt{9}$, which is 3.

$\therefore \sqrt{961}$ is either 31 or 39.

```
   31       39
 ×31      ×39
   31      351
  93      117
  961     1521
```

Since $31^2 = 961$,

$\therefore \sqrt{961} = 31$

Example 2

Find the square root of 625.

Hundreds place → 625 ← Units place

- Since the digit in the units place of 625 is 5, the digit in the units place of its square root is 5.
- Since the digit in the hundreds place of 625 is 6, the digit in the tens place of the square root, is the square root of the perfect square closest to 6 and less than it.
- The perfect square closest to 6 and less than it is 4. Its square root is 2.

$\therefore \sqrt{625}$ is 25.

Example 3

Find $\sqrt{784}$.

Method I

Hundreds place → 784 ← Units place

- Since the digit in the units place of 784 is 4, the digit in the units place of its square root is 2 or 8.
- Since the digit in the hundreds place of 784 is 7, the digit in the tens place of the square root, is the square root of the perfect square closest to 7 and less than it.
- The perfect square closest to 7 and less than it is 4. Its square root is 2.

$\therefore \sqrt{784}$ is either 22 or 28.

$$\begin{array}{r} 22 \\ \times 22 \\ \hline 44 \\ 44 \\ \hline 484 \end{array} \qquad \begin{array}{r} 28 \\ \times 28 \\ \hline 224 \\ 56 \\ \hline 784 \end{array}$$

$\therefore \sqrt{784} = 28$

Method II

The squares of the multiples of 10 which are less than 1000 are 100, 400 and 900. 784 lies between 400 and 900.

When written in order, we get;

$400 < 784 < 900.$

$\therefore \sqrt{400} < \sqrt{784} < \sqrt{900}$ (the square roots of these numbers)

That is, $20 < \sqrt{784} < 30$

Therefore, $\sqrt{784}$ lies between 20 and 30.

The digit in the units place of 784 is 4. So the digit in the units place of the square root is either 2 or 8. Therefore, $\sqrt{784}$ is either 22 or 28.

784 is closer to 900 than to 400.

As shown in the right $28^2 = 784$

$\therefore \sqrt{784}$ is 28.

Let us verify this.

$$\begin{array}{r} 28 \\ \times 28 \\ \hline 224 \\ 56 \\ \hline 784 \\ \hline \end{array}$$

Example 4

Show that 836 is not a perfect square.

Hundreds place → 836 ← Units place

- If 836 is a perfect square, then the digit in the units place of the square root should be 4 or 6.
- The digit in the hundreds place of 836 is 8. Since the closest perfect square less than 8 is 4, the digit in the tens place of the square root is $\sqrt{4}$, which is 2.

Therefore, if 836 is a perfect square, then its square root must be 24 or 26.

But $24 \times 24 = 576$ and $26 \times 26 = 676$. Therefore, 836 is not a perfect square.

Exercise 8.4

(1) Complete the table given below.

Perfect square	Square root of the perfect square
9	$\sqrt{9} = \sqrt{3^2} = 3$
36	
64	
121	
400	
900	

(2) Check whether the given numbers are perfect squares, and find the square root of each number which is a perfect square.

(i) 169 (ii) 972 (iii) 441 (iv) 716

(v) 361 (vi) 484 (vii) 1522 (viii) 529

(ix) 372 (x) 624

(3) The value of $\sqrt{324}$ is a whole number between 15 and 20. Find $\sqrt{324}$ by observing the last digit.

(4) 625 is a perfect square. Its square root is a whole number between 20 and 30. Find $\sqrt{625}$.

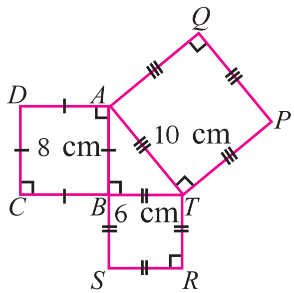
(5) Find the square root of each of the numbers given below by observation.

(i) 256 (ii) 441 (iii) 729 (iv) 361 (v) 841

Miscellaneous Exercise

(1) In the given figure, *ABCD* is a square of side length 8 cm, *BTRS* is a square of side length 6cm and *ATPQ* is a square of side length 10 cm.

(i) Find the area of the square *ABCD*.
(ii) Find the area of the square *BTRS*.
(iii) Find the area of the square *ATPQ*.
(iv) Find a special relationship between the areas of the three squares.



(2) The value of $\sqrt{500}$ cannot be found by using prime factors. Explain the reason for it.

(3) Show that $8^2 - 5^2 = (8 + 5)\ (8 - 5)$ and show the same relationship for another pair of perfect squares.

Summary

- We obtain a perfect square when we multiply a positive integer by itself.
- If a number is a square of a positive integer, then the square root of that number is that positive integer of which is the square.
- The symbol "$\sqrt{\ }$" is used to denote the square root of a positive number.
- The square root of a perfect square can be found by observing the last digit of that number.
- The square root of a perfect square can also be found using prime factors.

9 Mass

By studying this lesson, you will be able to,

- identify metric ton as a unit used to measure mass,
- know the relationship between kilogramme and metric ton.
- solve problems associated with mass which include metric tons.

9.1 Units used to measure mass

You have learnt before that milligramme, gramme and kilogramme are units used to measure mass. Now let us identify another unit used to measure mass.

It is mentioned that the mass of the paracetamol in a paracetamol tablet shown in the figure is 500 mg.

It is mentioned that the mass of the margarine in the packet of margarine shown in the figure is 250 g.

It is mentioned that the mass of the cement in the bag of cement shown in the figure is 50 kg.

The approximate mass of the lorry loaded with goods shown in the figure is mentioned as 20 t.

According to the information given above, in order to measure a heavy mass like a lorry, the unit metric ton is used, which is larger than the unit kilogramme (kg). The letter t is used to indicate "metric ton".

One metric ton is equal to a thousand kilogrammes. Accordingly, $1 \text{ t} = 1000 \text{ kg}$

The relationship between the above mentioned units used to measure mass is given below.

$$1 \text{ g} = 1000 \text{ mg}$$
$$1 \text{ kg} = 1000 \text{ g}$$
$$1 \text{ t} = 1000 \text{ kg}$$

9.2 The relationship between kilogramme and metric ton

• Expressing a mass given in metric tons in kilogrammes

Now let us see how to express a mass given in metric tons in kilogrammes.

Since $1 \text{ t} = 1000 \text{ kg}$

$2 \text{ t} = 2 \times 1000 \text{ kg} = 2000 \text{ kg}$

$3 \text{ t} = 3 \times 1000 \text{ kg} = 3000 \text{ kg}$

Accordingly, in order to express a mass given in metric tons in kilogrammes, the amount given in metric tons should be multiplied by 1000.

Example 1

Express 8.756 t in kilogrammes.

$8.756 \text{ t} = 8.756 \times 1000 \text{ kg}$

$= 8756 \text{ kg}$

Example 2

Express 3 t 850 kg in kilogrammes.

$3 \text{ t } 850 \text{ kg} = 3 \text{ t} + 850 \text{ kg}$

$= 3 \times 1000 \text{ kg} + 850 \text{ kg}$

$= 3000 \text{ kg} + 850 \text{ kg}$

$= 3850 \text{ kg}$

Example 3

Express 8.756 t in metric tons and kilogrammes.

$8.756 \text{ t} = 8 \text{ t} + 0.756 \text{ t}$

$= 8 \text{ t} + 0.756 \times 1000 \text{ kg}$

$= 8 \text{ t} + 756 \text{ kg}$

$= 8 \text{ t } 756 \text{ kg}$

Example 4

Express $3\frac{1}{2}$ t in kilogrammes.

$3\frac{1}{2} \text{ t} = 3 \text{ t} + \frac{1}{2} \text{ t}$

$= 3 \times 1000 \text{ kg} + 500 \text{ kg}$

$= 3000 \text{ kg} + 500 \text{ kg}$

$= 3500 \text{ kg}$

• Expressing a mass given in kilogrammes in metric tons

Next let us see how to express a mass given in kilogrammes in metric tons.

Since $1000 \text{ kg} = 1 \text{ t}$

$2000 \text{ kg} = \frac{2000}{1000} \text{ t} = 2 \text{ t}$

$3000 \text{ kg} = \frac{3000}{1000} \text{ t} = 3 \text{ t}$

Accordingly, in order to express a mass given in kilogrammes in metric tons, the amount given in kilogrammes should be divided by 1000.

Example 5

Express 2758 kg in metric tons.

$2758 \text{ kg} = \frac{2758}{1000} \text{ t}$

$= 2.758 \text{ t}$

Example 6

Express 2225 kg in metric tons and kilogrammes.

$2225 \text{ kg} = 2000 \text{ kg} + 225 \text{ kg}$

$= \frac{2000}{1000} \text{ t} + 225 \text{ kg}$

$= 2 \text{ t} + 225 \text{ kg}$

$= 2 \text{ t } 225 \text{ kg}$

When expressing a mass of 1000 kg or more in kilogrammes and metric tons, the number of kilogrammes is written as an addition of a multiple of 1000 and a number less than 1000.

Example 7

Express 3 t 675 kg in metric tons.

$3 \text{ t } 675 \text{ kg} = 3 \text{ t} + 675 \text{ kg}$

$= 3 \text{ t} + \frac{675}{1000} \text{ t}$

$= 3 \text{ t} + 0.675 \text{ t}$

$= 3.675 \text{ t}$

Example 8

Complete the table given below.

Mass	The mass in t and kg	The mass in metric tons
2400 kg	2 t 400 kg	2. 400 t
5850 kg	5 t 850 kg	5. 850 t
1050 kg	1 t 050 kg	1. 050 t
600 kg	0 t 600 kg	0. 600 t

Exercise 9.1

(1) Express the masses given below in metric tons.

(i) 2350 kg (ii) 5050 kg (iii) 3 t 875 kg (iv) 13 t 7 kg

(2) Express each mass given below in kilogrammes.

(i) 7 t (ii) 17 t (iii) 3 t 650 kg (iv) 2 t 65 kg

(v) 1.075 t (vi) 7.005 t (vii) 4.68 t (viii) $\frac{3}{4}$ t

(3) Express each mass given below in metric tons and kilogrammes.
(i) 1.275 t (ii) 2.025 t (iii) 5.75 t (iv) 7.3 t (v) 7.003 t

(4) The mass of a fully grown whale is approximately 19 000 kg. Express this mass in metric tons.

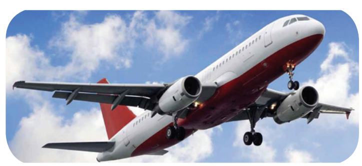
(5) Place a ✓ in front of the unit that is used to measure the mass of each item given below.

	The item to be measured	mg	g	kg and g	kg	t
(i)	A mango	....	....	....	....	....
(ii)	A comb of plantains	....	....	....	....	....
(iii)	A bag of sweet potatoes	....	....	....	....	....
(iv)	A loaf of bread	....	....	....	....	....
(v)	A lorry	....	....	....	....	....
(vi)	Ten travelling bags in a lift	....	....	....	....	....

(6) Complete the table given below.

The mass of the given item in metric tons	That mass in metric tons and kilogrammes	That mass in kilogrammes
1.6 t	1 t 600 kg	1600 kg
3.85 t	....	....
7.005 t	....	....
....	7 t 875 kg	....
....	6 t 5 kg	....
....	....	7008 kg
....	....	14 375 kg

9.3 Addition of two masses expressed in metric tons and kilogrammes



The total mass of the passengers and travelling bags in an air plane of mass 181 t 350 kg is 60 t 800 kg. Let us find the mass of the air plane with the passengers and travelling bags.

To do this, let us add the masses of the air plane, passengers and travelling bags.

Method 1

	t	kg
	181	350
+	60	800
	242	150

Let us add the quantities in the kilogrammes column.

350 kg + 800 kg = 1150 kg
1150 kg = 1000 kg + 150 kg
= 1 t + 150 kg

Let us write 150 kg in the kilogrammes column.

Let us carry 1 t to the metric tons column and add the quantities in this column.

1 t + 181 t + 60 t = 242 t

Let us write 242 t in the metric tons column.

Therefore, the total mass is 242 t 150 kg.

Method II

Let us express each mass in metric tons and then simplify.

181 t 350 kg = 181.350 t
60 t 800 kg = 60.8 t

181.350 t + 60.800 t = 242.150 t
242.150 t = 242 t + 150 kg

	t
	181 . 350
+	60 . 800
	242 . 150

Therefore, the total mass is 242 t 150 kg.

Method III

Let us express each mass in kilogrammes and simplify.

181 t 350 kg = 181 350 kg
60 t 800 kg = 60 800 kg
181 350 kg + 60 800 kg = 242 150 kg
242 150 kg = 242 t 150 kg

Therefore, the total mass is 242 t 150 kg.

Example 1

Add 10 t 675 kg and 3 t 40 kg.

	t	kg
	10	675
+	3	040
	13	715

Exercise 9.2

(1) Express the answer in metric tons and kilogrammes.

(i)

	t	kg
	2	780
+	1	620

(ii)

	t	kg
	3	450
	6	065
+	1	275

(iii) 10 t 225 kg + 6 t 705 kg

(iv) 150 t 650 kg + 40 t 460 kg

(2) The mass of a grown elephant is 4.75 t. The mass of a baby elephant is 2025 kg.

(i) Express the mass of the baby elephant in metric tons.
(ii) Find the total mass of both elephants in metric tons.
(iii) Express the total mass of both elephants in kilogrammes.

(3) A lorry of mass 3 t 450 kg is loaded with 2 t 700 kg of sugar and 4 t of rice. Find the total mass of the lorry with the goods loaded in it.

9.4 Subtraction of masses expressed in kilogrammes and metric tons

The total mass of a lorry loaded with rice is 10 t 250 kg. The mass of the lorry is 3 t 750 kg. Let us find the mass of the rice loaded in the lorry.

In order to find the mass of the rice loaded in the lorry, the mass of the lorry should be subtracted from the total mass.

Method I

	t	kg
	10	250
−	3	750
	6	500

Since 750 kg cannot be subtracted from 250 kg, let us carry 1 t from the 10 t in the metric tons column, that is, 1000 kg, to the kilogrammes column and add it to the 250 kg in the kilogrammes column.
Then, 1000 kg + 250 kg = 1250 kg.
1250 kg − 750 kg = 500 kg
Let us write 500 kg in the kilogrammes column.

Let us subtract 3 t from the remaining 9 t in the metric tons column.
Then, 9 t − 3 t = 6 t
Let us write 6 t, in the metric tons column.

Therefore, the mass of the rice is 6 t 500 kg.

Method II

Let us express each mass in metric tons and then simplify.

$10\ t\ \ 250\ kg = 10.250\ t$

$3\ t\ \ 750\ kg = 3.750\ t$

$10.250\ t - 3.750\ t = 6.500\ t$

$6.500\ t = 6\ t\ \ 500\ kg$

$$\begin{array}{r} \text{t} \\ 10\,.\,250 \\ -\ 3\,.\,750 \\ \hline 6\,.\,500 \\ \hline \hline \end{array}$$

The mass of the rice in the lorry is 6 t 500 kg.

Method III

Let us express each mass in metric tons and then simplify.

$10\ t\ \ 250\ kg = 10\ 250\ kg$

$3\ t\ \ 750\ kg = 3750\ kg$

$10\ 250\ kg - 3750\ kg = 6500\ kg$

$6500\ kg = 6\ t\ \ 500\ kg$

$$\begin{array}{r} \text{kg} \\ 10\ 250 \\ -\ 3\ 750 \\ \hline 6\ 500 \\ \hline \hline \end{array}$$

The mass of the rice in the lorry is 6 t 500 kg.

Exercise 9.3

(1) Subtract the following.

(i)
$$\begin{array}{rrr} & \text{t} & \text{kg} \\ & 5 & 000 \\ - & 2 & 750 \\ \hline \hline \end{array}$$

(ii)
$$\begin{array}{rrr} & \text{t} & \text{kg} \\ & 4 & 350 \\ - & 1 & 650 \\ \hline \hline \end{array}$$

(iii) 250 t 650 kg – 150 t 105 kg

(iv) 60 t – 25 t 150 kg

9.5 Multiplication of a mass expressed in metric tons and kilogrammes by a number

- The mass of a concrete beam used to build a flyover bridge is 6 t 500 kg. Five such beams are placed across two columns. Let us find the total mass borne by the two columns.

The two columns bear 5 beams of mass 6 t 500 kg each. Hence, in order to find the mass borne by the two columns, 6 t 500 kg should be multiplied by 5.

Method I

Let us express 6 t 500 kg in kilogrammes and then multiply it by 5.

6 t 500 kg = 6500 kg

6500 kg × 5 = 32 500 kg

kg
6500
×5
32 500

32 500 kg = 32 t 500 kg

Accordingly, the total mass borne by the two columns is 32 t 500 kg.

Method II

t	kg
6	500
×	5
32	500

First, let us multiply 500 kg by 5.

500 × 5 kg = 2500 kg

2500 kg = 2000 kg + 500 kg = 2 t + 500 kg

Let us write 500 kg in the kilogrammes column.

Let us multiply 6 t by 5. 6 t × 5 = 30 t

Now let us add the 2 t obtained by the multiplication in the kilogrammes column, to the 30 t in the metric tons column.

30 t + 2 t = 32 t

Let us write 32 t in the metric tons column.

➢ Let us simplify 5 t 120 kg × 12.

Method I

t	kg
5	120
×	12
61	440

First let us multiply 120 kg by 12.

120 kg × 12 = 1440 kg = 1 t 440 kg

Now let us multiply 5 t by 12.

5 t × 12 = 60 t

∴ 5 t 120 kg × 12 = 60 t + 1 t 440 kg

= 60 t + 1 t + 440 kg

= 61 t 440 kg

5 t 120 kg × 12 = 61 t 440 kg

Method II

Let us express 5 t 120 kg in kilogrammes and multiply it by 12.

5 t 120 kg = 5120 kg

Let us multiply 5120 kg by 12.

5 120 kg × 12 = 61 440 kg

= 61 t 440 kg

kg
5120
× 12
10240
5120
61440

Example 1

(1) The mass of a tin of milk powder is 500 g. The mass of the empty tin is 50 g.

(i) Find the mass of the milk powder in such a tin, in grammes. Express this mass in kilogrammes.

(ii) A container is loaded with 1000 such tins of milk powder. Write the mass of these 1000 tins in kilogrammes and express it in metric tons also.

(i) The mass of a tin of milk powder = 500 g

The mass of the milk powder in the tin = 500 g – 50 g = 450 g

= 450 ÷ 1000 kg = 0.45 kg

(ii) The mass of 1000 tins of milk powder = 500 × 1000 g = 500 000 g

= 500 000 ÷ 1000 kg = 500 kg

= 500 ÷ 1000 t = 0.5 t

Exercise 9.4

(1) Simplify the following.

(i)

t	kg
160	200
	× 5

(ii)

t	kg
165	465
	× 4

(iii)

t	kg
32	45
	× 3

(iv) 16 t 325 kg × 12 (v) 5 t 450 kg × 25 (vi) 64.5 t × 50

(vii) 27.3 t × 25

(2) (i) The approximate mass of a car is 1 t 200 kg. Express the approximate mass of 10 such cars in metric tons.

(ii) The mass of a vehicle which transports these 10 cars is 20 t. Accordingly, express the total mass of the vehicle with these 10 cars, in metric tons.

9.6 Division of a mass by a whole number

- If a mass of 6 t 750 kg of rice is loaded equally into 5 lorries, let us find the mass of the rice loaded into one lorry. For this, 6 t 750 kg should be divided by 5.

Method 1

	t	kg
	1	350
5	6	750
	5	
	1 →	1000
		1750
		1750
		0000

First, let us divide the metric ton quantity.
Since there is 1, 5s in 6, let us write 1 in the relevant position of the metric tons column where the answer should be written, and carry the remaining 1 t to the kilogrammes column as 1000 kg.

Next let us find the amount of kilogrammes in the kilogrammes column.
1000 kg + 750 kg = 1750 kg
Let us divide 1750 kg, by 5.
(1750 kg ÷ 5 = 350 kg)

The mass of the rice loaded into one lorry is 1 t 350 kg.

Method II

Let us express 6 t 750 kg in kilogrammes and divide it by 5.

6 t 750 kg = 6750 kg
6750 kg ÷ 5 = 1350 kg

	kg
	1350
5	6750
	5
	17
	15
	25
	25
	000
	000
	0

The mass of the rice loaded into one lorry is 1350 kg.

- A mass of 16 t 200 kg of paddy in a storehouse is loaded equally into 9 lorries. Let us find the mass of the paddy loaded into one lorry.

For this, 16 t 200 kg should be divided by 9.

Method I

```
      t      kg
      1     800
   ┌──────────────
 9 │ 16     200
     9
   ├──────────────
   │ 7 → 7000
         7200
         7200
         ----
         0000
```

Let us divide the 16 t in the metric tons column by 9.
Let us carry the remaining 7 t to the kilogrammes column as 7000 kg.
Now let us find the amount of kilogrammes in the kilogrammes column.

7000 kg + 200 kg = 7200 kg

Let us divide 7200 kg by 9.

7200 kg ÷ 9 = 800 kg

The mass of the paddy loaded into one lorry is 1 t 800 kg.

Method II

Let us express 16 t 200 kg in kilogrammes and divide by 9.

16 t 200 kg = 16 t + 200 kg
= 16 000 kg + 200 kg
= 16 200 kg

16 200 kg ÷ 9 = 1800 kg

```
       kg
     1 800
   ┌─────────
 9 │ 16 200
     9
   ├─────────
     72
     72
   ──────
      00
      00
       00
       00
```

1800 kg = 1 t 800 kg

The mass of the paddy in one lorry is 1 t 800 kg.

Example 1

A lorry had to make 7 trips in order to transport a quantity of rice of mass 66.5 t. If the lorry carried an equal amount of rice on each trip, find the mass of the rice it carried on one trip.

```
        t
       9.5
  7 | 66.5
    | 63
    |  3 5
    |  3 5
    |    0
```

The mass of the rice carried by the lorry on 7 trips = 66.5 t

The mass of the rice carried by the lorry on one trip = 66.5 t ÷ 7

= 9.5 t

Exercise 9.5

(1) Simplify the following.

(i) 5 t 200 kg ÷ 4 (ii) 12 t ÷ 5 (iii) 14 t 500 kg ÷ 5

(iv) 15 t ÷ 200 (v) 3 t ÷ 40 (vi) 17 t 300 kg ÷ 8

Summary

- Milligramme (mg), gramme (g), kilogramme (kg) and metric ton (t) are some units used to measure mass.

  1 g = 1000 mg 1 kg = 1000 g 1 t = 1000 kg
- In order to express a mass given in metric tons in kilogrammes, the quantity given in metric tons needs to be multiplied by 1000.
- In order to express a mass given in kilogrammes in metric tons, the quantity given in kilogrammes needs to be divided by 1000.

10 Indices

By studying this lesson you will be able to,

- express a power of a product as a product of powers,
- express a product of powers as a power of a product, and
- find the value of a power of a negative integer by expansion.

10.1 Indices

Let us recall what was learnt in Grade 7 about indices.

You learnt in Grade 7 that 2^3 and x^4 are respectively a power of 2 and a power of x. In 2^3, the base is 2 and the index is 3.

These can be expanded and written as products as $2^3 = 2 \times 2 \times 2$ and $x^4 = x \times x \times x \times x$.

Accordingly, $3x^2y^3 = 3 \times x \times x \times y \times y \times y$ and $3ab = 3 \times a \times b$.
Since $6 = 2 \times 3$, we say that 6 is the product of 2 and 3.
Similarly, since $3ab = 3 \times a \times b$, we say that $3ab$ is the product of 3, a and b.

Do the following review exercise to recall the facts that have been learnt so far regarding indices.

Review Exercise

(1) Complete the following table.

Number	Index Notation	Base	Index
8	2^3	....	....
9	....	....	....
16	....	2	....
....	....	4	2
1000	....	10	....

(2) Expand and write each of the following expressions as a product.

(i) $3x^2$ (ii) $2p^2q$ (iii) $4^2\,x^3$ (iv) $5^2\,x^2\,y^2$

(3) Write down each of the following numbers as a product of powers of which the bases are prime numbers.

(i) 20 (ii) 48 (iii) 100 (iv) 144

(4) Write 64 in index notation (i) with base 2
(ii) with base 4
(iii) with base 8.

10.2 Expressing a power of a product as a product of powers

2×3 is the product of 2 and 3. $(2 \times 3)^2$ is a power of the product 2×3.
Let us write $(2 \times 3)^2$ as a product of powers of 2 and 3.

$$\begin{aligned}(2 \times 3)^2 &= (2 \times 3) \times (2 \times 3)\\ &= 2 \times 3 \times 2 \times 3\\ &= 2 \times 2 \times 3 \times 3\\ &= 2^2 \times 3^2\end{aligned}$$

$\therefore\ (2 \times 3)^2 = 2^2 \times 3^2$

Now let us write $(2 \times 3)^3$ as a product of powers of 2 and 3.

$$\begin{aligned}(2 \times 3)^3 &= (2 \times 3) \times (2 \times 3) \times (2 \times 3)\\ &= 2 \times 3 \times 2 \times 3 \times 2 \times 3\\ &= 2 \times 2 \times 2 \times 3 \times 3 \times 3\\ &= 2^3 \times 3^3\end{aligned}$$

$\therefore\ (2 \times 3)^3 = 2^3 \times 3^3$

Accordingly, the power of a product can be written in this manner as a product of powers of the factors of the given product.

Now let us consider a power of a product that contains unknown terms.

$$\begin{aligned}(ab)^3 &= ab \times ab \times ab\\ &= a \times b \times a \times b \times a \times b\\ &= a \times a \times a \times b \times b \times b\\ &= a^3 \times b^3 = a^3\,b^3\end{aligned}$$

$(ab)^3 = a^3b^3$

Let us similarly express $(abc)^3$ as a product of powers of a, b and c.

$$(abc)^3 = (abc) \times (abc) \times (abc)$$
$$= (a \times b \times c) \times (a \times b \times c) \times (a \times b \times c)$$
$$= (a \times a \times a) \times (b \times b \times b) \times (c \times c \times c)$$
$$= a^3 \times b^3 \times c^3 = a^3\, b^3\, c^3$$

$$\therefore\ (abc)^3 = a^3b^3c^3$$

Accordingly, a power of a product can be written as a product of powers of the factors of the given product.

- Let us express $4a^2$, as a power of a product.

$$4a^2 = 4 \times a^2 = 2^2 \times a^2$$
$$= (2 \times a)^2$$
$$= (2a)^2$$

This can be established further through the following examples.

Example 1

Express each of the following powers of products as a product of powers of the factors of the given product.

(i) $(2x)^3$ (ii) $(3ab)^2$

(i) $(2x)^3 = 2^3 \times x^3$
$= 2^3x^3$

(ii) $(3ab)^3$
$(3ab)^3 = 3^3 \times a^3 \times b^3$
$= 3^3a^3b^3$

Example 2

Express $36x^2$, as a power of a product.

Since $36 = 6^2$,

$$36x^2 = 6^2 \times x^2 = (6 \times x)^2$$
$$= (6x)^2$$

Example 3

Express a^3b^3 as a power of a product.

$$a^3b^3 = a^3 \times b^3$$
$$= (a \times b)^3$$
$$= (ab)^3$$

Exercise 10.1

(1) Express each of the following powers of products as a product of powers of the factors of the given product.

(a) (i) $(2 \times 5)^2$ (ii) $(3 \times 5)^3$ (iii) $(11 \times 3 \times 2)^3$
(iv) $(a \times b)^2$ (v) $(x \times y)^5$ (vi) $(4 \times x \times y)^3$

(b) (i) $(5a)^2$ (ii) $(6p)^2$ (iii) $(4y)^3$
(iv) $(3a)^3$ (v) $(2y)^4$ (vi) $(2ab)^2$

(2) Find the value of each of the following powers of products. Write each power of a product as a product of powers of the factors of the given product and obtain the value again by simplifying the answer.

(i) $(2 \times 5)^3$ (ii) $(2 \times 3)^3$ (iii) $(11 \times 2)^3$

(iv) $(3 \times 7)^2$ (v) $(5 \times 7)^3$ (vi) $(13 \times 2 \times 3)^2$

(3) Express each of the following products of powers as a power of a product.

(i) $5^2 \times 2^2$ (ii) $5^2 \times 11^2$ (iii) $3^3 \times 4^3 \times 2^3$

(iv) $x^2 \times y^2$ (v) $p^3 \times q^3$ (vi) $a^5 \times b^5 \times x^5$

(vii) $100\ m^2$ (viii) $225\ t^2$ (ix) $8\ y^3$

(4) Show that $1000x^3 = (10x)^3$.

10.3 The power of a negative integer

-1, -2, -3 are negative integers. Do the following activity to find the value of a power of these negative integers.

Activity 1

Complete the following table by using the knowledge on multiplying integers.

Integer	Its second power	Its third power	Its fourth power
2	$2^2 = 2 \times 2 = 4$	$2^3 = 2 \times 2 \times 2 = 8$	$2^4 = 2 \times 2 \times 2 \times 2 = 16$
-1	$(-1)^2 = (-1) \times (-1) = 1$	....	....
-2	....	....	....
-3	....	....	....

- the value of any power of a positive integer is positive.
- the value of an odd power of a negative integer is negative.
- the value of an even power of a negative integer is positive.

Example 1

Find the value of $(-2)^4$.

$$(-2)^4 = 2^4$$
$$= 16$$

Example 2

Find the value of $(-5)^3$.

$$(-5)^3 = -(5)^3$$
$$= -125$$

Exercise 10.2

(1) Find the value.

(a) (i) $(-1)^1$ (ii) $(-1)^2$ (iii) $(-1)^3$ (iv) $(-1)^4$
(v) 1^1 (vi) 1^{1003} (vii) 1^{2018} (viii) 1^0

(b) (i) $(-4)^2$ (ii) $(-4)^3$ (iii) $(-4)^4$ (iv) $(-5)^1$
(v) $(-5)^2$ (vi) $(-5)^3$ (vii) $(-10)^1$ (viii) $(-10)^2$

(2) Show that $(-1)^8 > (-1)^9$.

Miscellaneous Exercise

(1) Express each of the following products of powers as a power of a product.

(i) $(2x)^2 \times y^2$ (ii) $(3a)^2 \times b^2$ (iii) $p^3 \times (2q)^3$
(iv) $(2x)^3 \times (3y)^3$ (v) $(5a)^3 \times (2b)^3$ (vi) $a^3 \times (2b)^3 \times c^3$

(2) Show that $(3a)^2 \times (2x)^2 = 36a^2x^2$.

(3) Arrange in ascending order of the values.
(i) 2^3, $(-10)^1$, $(-1)^{10}$, 3^2
(ii) $(-2)^4$, $(-2)^5$, $(-1)^4$, $(-1)^5$

(4) If a is a negative integer, show that $a^2 > a^3$.

Summary

- If a, b, c and n are positive integers, then $(ab)^n = a^n \times b^n = a^n b^n$ and $(abc)^n = a^n \times b^n \times c^n = a^n b^n c^n$.
- The value of any power of a positive integer is positive.
- The value of an odd power of a negative integer is negative.
- The value of an even power of a negative integer is positive.

REVISION EXERCISE – FIRST TERM

(1) (i) Find the value of $\sqrt{361}$.

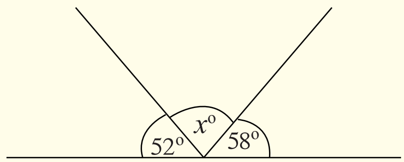
(ii) Evaluate 5t 75 kg × 12.

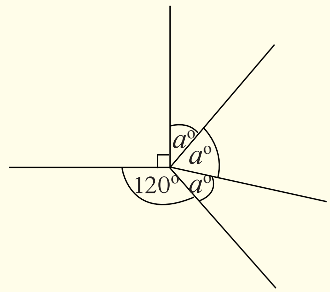
(iii) Write down the value of $(-11)^{11}$.

(iv) What is the complement of the angle 28^{o} ?

(v) What is the supplement of the angle 28^{o} ?

(vi) (*a*) Find the value of *x*. (*b*) Find the value of *a* .





(vii) Write down the number of faces, edges and vertices of a dodecahedron.

(viii) Fill in the blanks.

$12x - 36y + 4 = 4\ (\square\ x - \square\ y + \square\ )$

(2) (*a*) Find the value of each of the following.

(i) (– 5) + (– 3) (ii) (– 7) + 4 (iii) 13 + (– 5)

(iv) (– 5) – (– 2) (v) (– 7) – (– 10) (vi) 0 – (– 5)

(*b*) Find the value of each of the following.

(i) (– 12) × (– 3) (ii) (+ 8) × (– 5) (iii) (+ 12) ÷ (– 3)

(iv) (– 12) ÷ (– 3) (v) (– 12) × 0 (vi) 0 ÷ (– 100)

(*c*) Fill in the cages and re-write the following.

(i) 24 ÷ $\square$ = (– 4) (ii) (– 16) ÷ $\square$ = (– 4) (iii) 32 ÷ $\square$ = (– 4)

(iv) (– 10) + $\square$ = – 6 (v) (– 5) + $\square$ = (– 6) (vi) (– 2) × (– 4) = $\square$

(3) The general term of the triangular number pattern starting from 1 is $\dfrac{n\,(n+1)}{2}$.

(i) Write the first term of the triangular number pattern.

(ii) Write the 19th and 20th terms of this pattern.

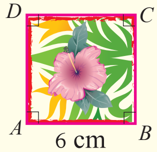
(iii) It is given that $10 \times 11 = 110$. Find which term of the triangular number pattern is 55?

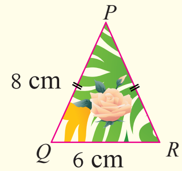
(iv) It is given that $18 \times 19 = 342$. Find which term of the triangular number pattern is 171?

(v) Show that the sum of the 19th and 20th terms of the triangular number pattern is equal to the 20th term of the square number pattern which starts from 1.

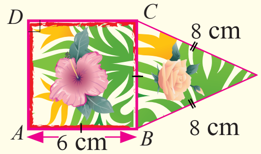
(4) (i) Find the perimeter of the square design.

(ii) Find the perimeter of the isosceles triangular design.

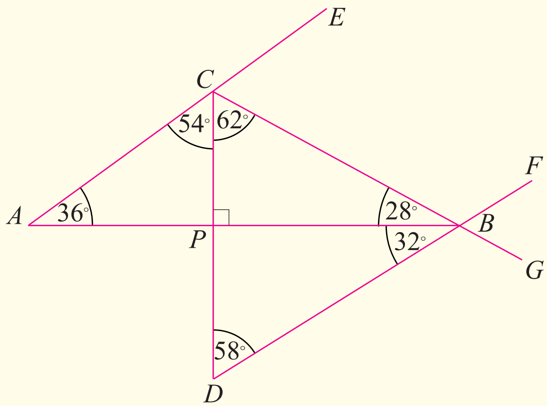




(iii) The two designs are pasted together as shown to form a composite figure. Find the perimeter of the composite figure.



(5)



The straight lines AB and CD are drawn such that they intersect each other perpendicularly at P. The given figure is obtained by joining AC, CB and DB and then producing them.

(i) Write three pairs of complementary angles.

(ii) Write three pairs of supplementary angles.

(iii) Write 4 pairs of vertically opposite angles.

(iv) What is the magnitude of $F\hat{B}G$?

(v) $C\hat{B}D$ and $D\hat{B}G$ are a pair of supplementary angles. Write down the magnitude of $D\hat{B}G$.

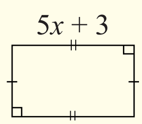
(vi) Name an angle which is a supplement of $C\hat{B}P$.

(vii) Write down the value of the angle you named.

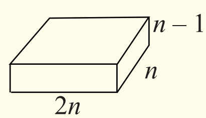
(viii) Find the magnitude of $C\hat{B}F$.

(ix) Find the sum of the angles around the point B and establish the fact that the sum of the angles around a point is 360^0.

(6) (i) The perimeter of a rectangle is $16x + 10$ units. Its length is $5x + 3$ units. Write an algebraic expression for its breadth.

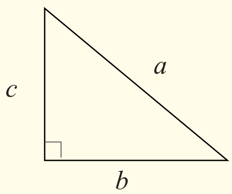


(ii) A cuboid of length, breadth and height equal to $2n$, n and $n - 1$ units respectively is shown in the figure. Show that the sum of the lengths of all its edges is $4\,(4n - 1)$ units.



(7) Simplify the following.

(i) $5(c-2)+12$ (ii) $7(d-9)-d$
(iii) $4(f+5)+2f-3$ (iv) $-2g(h+4)-3g(h-2)$
(v) $4h(i+2)-7(i-1)$

(8) 

If the relationship $a^2 = b^2 + c^2$ is true for this right angled triangle, find a when $b = 8$ cm and $c = 6$ cm.

(9) (i) Express $4y^2$ as a power of a product.
(ii) Write $(8ab)^2$ as a product of powers and simplify it.
(iii) Simplify $(2p)^3 \times (3p)^3$.
(iv) Show that 6^3 is equal to 8×27.
(v) Show that when $(-3)^4$ is simplified, the same value as of 9^2 is obtained .
(vi) Without obtaining the value of $(-15)^3 \times (-27)^4$, explain whether the answer is a positive value or a negative value.

(10) A signboard near an old bridge indicates that the maximum load which the bridge can support is 8 t. A lorry of mass 5.5 metric tons is loaded with 80 bags of cement of mass 50 kg each.

(i) Calculate and show that it is dangerous for the lorry loaded with the bags of cement to cross the bridge.
(ii) What is the minimum number of bags of cement that should be removed for the lorry to safely cross the bridge?

(11) Simplify the following.

(a)	(b)	(c)
(i) $(+7)+(-3)$	(i) $(+10)-(-3)$	(i) $(+4)\times(-3)$
(ii) $(-5)+(-4)$	(ii) $(-7)-(-3)$	(ii) $(-5)\times(-6)$
(iii) $(+12)+(-18)$	(iii) $(-7)-(+20)$	(iii) $(-1)\times(+4.8)$
(iv) $(+5\frac{1}{2})+(-3)$	(iv) $(+17)-(-12)$	(iv) $(-20)\div(+4)$
(v) $(+3.7)+(-6.3)$	(v) $(+8.7)-(-2.3)$	(v) $(-35)\div(-5)$

(12) Expand the given algebraic expressions and simplify them.

(i) $5(2x-3)-4x+7$ (ii) $x(3y+5)-8xy+2$ (iii) $-3a(5-7b)+5(a-2)$

(13) Simplify the following.

(i) $4a + 7b - 3(a + c)$

(ii) $2(3x - 7) - 2x + 5$

(iii) $3a(a + 7) + 5a^2 - 20a + 4$

(14) Find the value of each algebraic expression when $x = -2$, $y = 3$ and $z = -3$.

(i) $3x + 4y$ (ii) $x^2y + 5y^2$ (iii) $4(2x - 3y - 4z)$

(15) Write down the geometrical shape of the faces of each solid given below.

(i) Regular tetrahedron

(ii) Cube

(iii) Regular octahedron

(iv) Regular dodecahedron

(v) Regular icosahedron

(16) Write down the HCF of each of the following groups of terms.

(i) $3x, 12xy, 15y$ (ii) $12x, 6xy, 9x^2$

(iii) $3a^2b, 15ab, 15y$ (iv) $4x^2y, 6xy, 8xy^2$

(17) Factorize the following expressions.

(i) $8x + 4y + 12$ (ii) $15x^2 + 3xy$

(iii) $6a^2b - 15ab + 18abc$ (iv) $-4mn - 20m^2 + 12m$

(18) (i) Write down the perfect squares that are in the range of values from 1 to 100.

(ii) The digit in the units place of a perfect square is 6. Write two digits that could be in the units place of its square root.

(iii) Which digits do not appear in the units place of a perfect square?

(iv) Find $\sqrt{900}$.

(19) Fill in the blanks.

(i) 3 t = ... kg. (ii) 3500 kg = ... t ....kg.

(iii) 4.05 t = ...kg. (iv) 12 450 kg = ... t.

(v) 10 t 50 kg = ...kg.

(20) Evaluate the following.

(i) $3^2 \times 5$ (ii) $4^3 \times 2^2$ (iii) $2^3 \times 3^2$

(iv) $(-4)^2 \times 5^3$ (v) $(-3)^3 \times 2^2$ (vi) $(-1)^4 \times 5^2 \times 4$

11 Symmetry

By studying this lesson you will be able to,

- identify the rotational symmetry of a plane figure,
- find the order of rotational symmetry of a plane figure that has rotational symmetry, and
- find the relationship between the number of axes of bilateral symmetry and the order of rotational symmetry of a plane figure which is bilaterally symmetrical.

11.1 Bilateral symmetry

You learnt in Grade 7 that if it is possible to fold a plane figure through a line on it to get two identical parts which coincide with each other, then it is called a **bilaterally symmetrical plane figure**. You also learnt that such a line of a bilaterally symmetrical figure is known as an **axis of symmetry** of the figure.

The two parts on either side of an axis of symmetry of a bilaterally symmetrical figure are equal in shape and area.

If by folding a plane figure along a line, it is divided into two parts which are equal in area and shape, but the two parts do not coincide with each other, then that line is not an axis of symmetry of the plane figure.

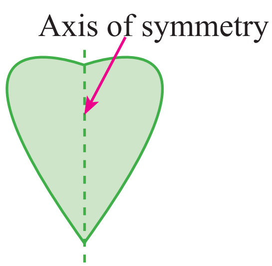


Figure 1

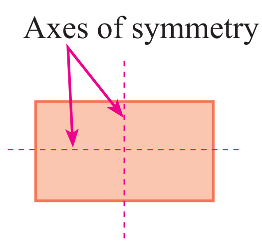


Figure 2

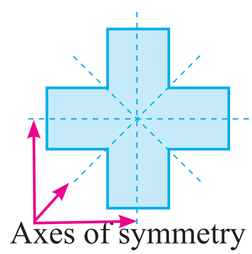


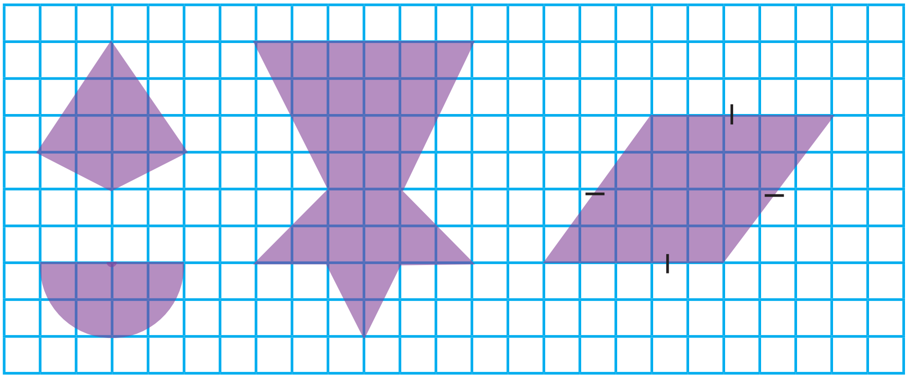
Figure 3

The axes of symmetry are indicated by dotted lines in each figure shown above.

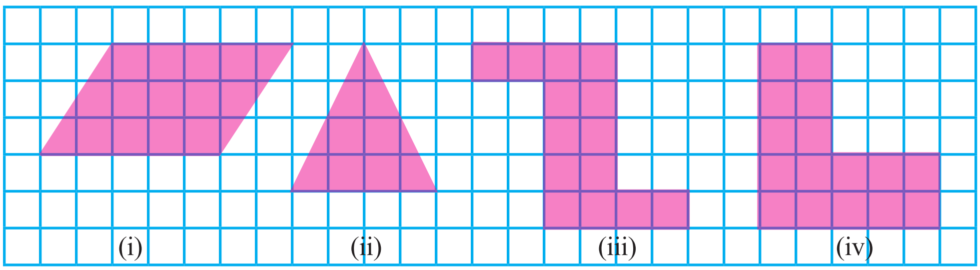
Do the review exercise to recall the facts you learnt in Grade 7 about bilateral symmetry.

Review Exercise

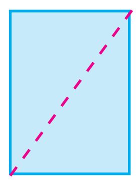
(1) Copy the following plane figures in your exercise book and draw the axes of symmetry of each figure.



(2) Select the figures that are bilaterally symmetrical from the following figures and write down their numbers.



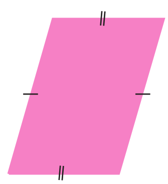
(3) The dotted line shown in the figure divides the given rectangle into two equal parts. Samith says that the dotted line represents an axis of symmetry of the rectangle. Explain why his statement is not true.



(4) (i) Copy the given parallelogram onto a tracing paper and cut it out.

(ii) Can the figure that was cut out be folded along a line so that the two parts on either side coincide?

(iii) Accordingly, show that a parallelogram need not have bilateral symmetry.



11.2 Rotational symmetry

When a plane figure is rotated about a point on it through one complete rotation in the plane of the figure, it coincides with the original position at least once.

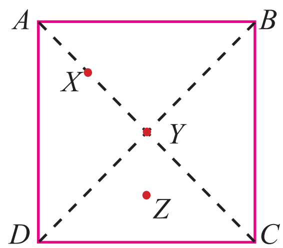
There are some figures, which when rotated about a point on it through one complete rotation, coincide several times with the original position.

The number of times a figure coincides with the original position varies, depending on the point about which it is rotated.

Do the following activity to establish this property further.

Activity 1

Step 1 - Draw a square *ABCD* and mark the points *X*, *Y* and *Z* as in the given figure.



Step 2 - Trace the figure *ABCD* onto a transparent sheet like an oil paper or a plastic paper and mark the points *X*, *Y* and *Z* on it.

Step 3 - Overlap the two figures so that they coincide, and keep them in place with a pin fixed at point *X*.

Step 4 - Rotate the figure on the transparent sheet about the pin point (point *X*) and observe whether the two figures coincide with each other. Find the number of times the two figures coincide with each other when the transparent sheet is rotated once about *X*.

Step 5 - As above, rotate the figure on the transparent sheet once about the points *Y* and *Z* too, and find the number of times the two figures coincide with each other.

Step 6 - Draw the following table in your exercise book and complete it.

Point	*X*	*Y*	*Z*
Number of times the figures coincide			

While doing the above activity you would have observed that when the figure on the transparent sheet was rotated once about the points X and Z, the two figures coincided with each other only once at the completion of one rotation, and that when the figure was rotated about the point Y, it coincided four times with the original figure during one complete rotation.

When a plane figure is rotated about a fixed point in it through one complete rotation (i.e., 360°), if it coincides with the original position before completing one rotation, then it is said to have **rotational symmetry**. The point about which the figure is rotated is called the **centre of rotation**.

When a plane figure that has rotational symmetry is rotated once about a point which is not the centre of rotation, then it coincides with the original figure only when it completes one complete rotation.

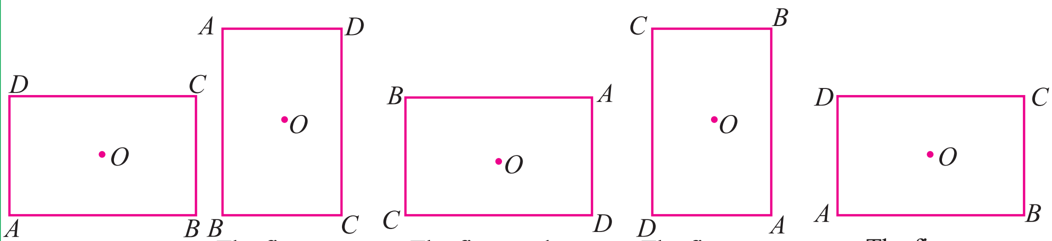
The number of times a figure that has rotational symmetry coincides with itself when it is rotated once about the centre of rotation is called its **order of rotational symmetry**.

From the above activity, it is clear that a square has rotational symmetry, that its centre of rotation is the point at which its axes of symmetry intersect, and that its order of rotational symmetry is 4.

Activity 2

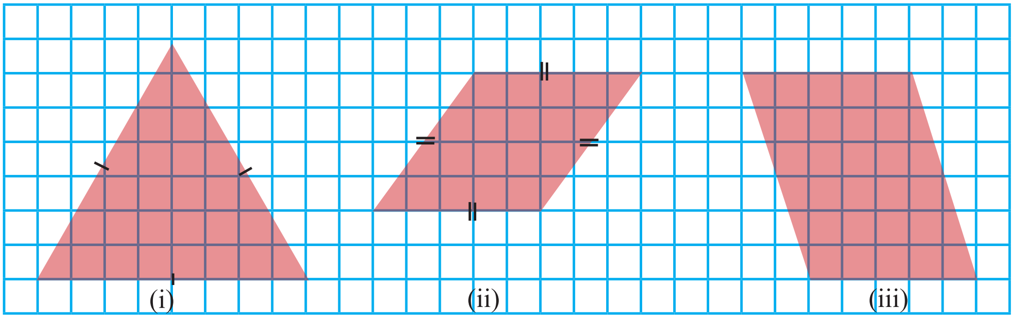
Step 1 - Draw a rectangle in your exercise book and name it $ABCD$.

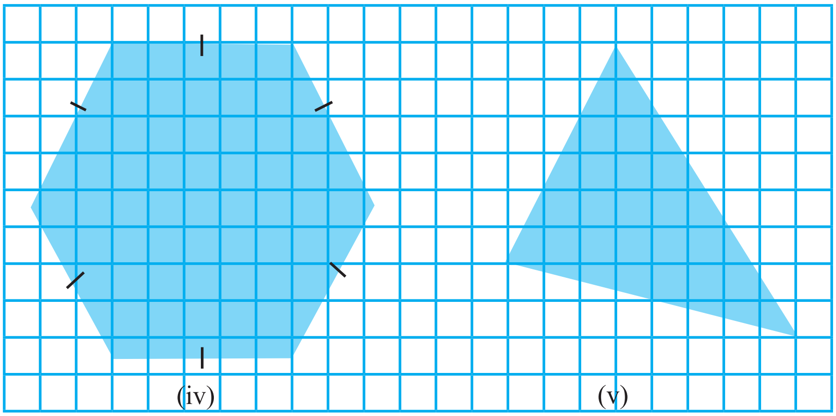
Step 2 - Copy the rectangle $ABCD$ onto a plastic paper and place it on the original figure so that they coincide with each other. By using a pin, rotate the plastic paper about O as in activity 1 and find out whether the rectangle has rotational symmetry. If it does, find its order of rotational symmetry.



The figure when rotated through 90° | The figure when rotated through 180° | The figure when rotated through 270° | The figure when rotated through 360°

Step 3 - Draw the following figures also in your exercise book, and check whether they have rotational symmetry.





Step 4 - Copy the following table and complete it.

If a given figure has rotational symmetry, then write its order of symmetry.

Plane figure	Number of axes of symmetry	Order of rotational symmetry
Rectangle		
Equilateral triangle		
Rhombus		
Parallelogram		
Regular hexagon		
Scalene triangle		

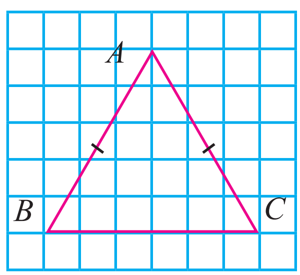
Examine the table given below.

Plane figure	**Number of axes of symmetry**	**Order of rotational symmetry**	**Rotational symmetry Yes/No**
Equilateral triangle	3	3	Yes
Parallelogram	0	2	Yes
Rhombus	2	2	Yes
Rectangle	2	2	Yes
Square	4	4	Yes
Regular pentagon	5	5	Yes
Regular hexagon	6	6	Yes
Regular octagon	8	8	Yes

The following facts are clear according to the above table.

- If a **geometrical plane figure** which has rotational symmetry is also **bilaterally symmetrical**, then its order of rotational symmetry is equal to the number of axes of bilateral symmetry.
- A figure which is not bilaterally symmetrical can have rotational symmetry (parallelogram).
- The center of rotational symmetry of a bilaterally symmetrical plane figure which has rotational symmetry, is the point of intersection of its axes of bilateral symmetry.
- If the order of rotational symmetry of a plane figure is 2 or more, then that figure is said to have rotational symmetry.
- The order of rotational symmetry of a figure that has rotational symmetry is greater than 1.

Exercise 11.1



(1) (i) Draw the isosceles triangle ABC in your exercise book as in the figure, and draw its axis of symmetry.

(ii) Copy the triangle ABC onto a plastic paper and find out whether an isosceles triangle has rotational symmetry.

(iii) Does every plane figure which is bilaterally symmetrical have rotational symmetry?

(2) (i) Draw a plane figure which has two or more axes of symmetry.

(ii) Find out whether the figure you have drawn has rotational symmetry.

(iii) If it has rotational symmetry, mark the centre of rotation as P and write down the order of rotational symmetry of the figure.

(3) Write down the following statements in your exercise book and mark the true statements with "✓" and the false statements with "×".

(i) Every bilaterally symmetrical plane figure has rotational symmetry.

(ii) Every plane figure which has rotational symmetry is also bilaterally symmetrical.

(iii) If a bilaterally symmetrical plane figure has rotational symmetry, then the number of axes of symmetry is equal to its order of rotational symmetry.

(iv) The point of intersection of the axes of symmetry of a bilaterally symmetrical plane figure which has more than one axis of symmetry is its centre of rotational symmetry.

(v) A scalene triangle does not have rotational symmetry and is also not bilaterally symmetrical.

Summary

- When a plane figure is rotated about a special point in it through an angle of 360^{o}, if it coincides with the original position before completing a full rotation, then it is said to have rotational symmetry.
- The center of rotational symmetry of a bilaterally symmetrical plane figure which has rotational symmetry, is the point of intersection of its axes of bilateral symmetry.
- The order of rotational symmetry of a figure that has rotational symmetry is greater than 1.
- When a plane figure is rotated about its centre of rotation, the number of times it coincides with the original position during a complete rotation is called its order of rotational symmetry.

12 Triangles and Quadrilaterals

By studying this lesson, you will be able to,

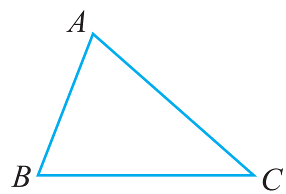
- obtain a value for the sum of the interior angles of a triangle and of a quadrilateral,
- show that the sum of the exterior angles of a triangle and of a quadrilateral is 360^{o}, and
- perform calculations associated with angles of triangles and quadrilaterals.

12.1 Triangles

You have learnt that a polygon formed with three straight line segments is called a **triangle.**

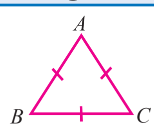
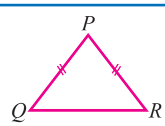
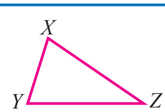
A triangle has three sides and three angles. They are called the elements of the triangle.

The three sides of the triangle ABC are AB, BC and CA. The three angles of the triangle ABC are $A\hat{B}C$, $B\hat{C}A$ and $C\hat{A}B$.

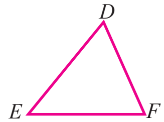
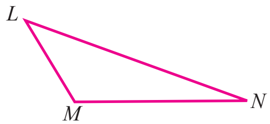
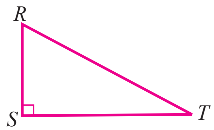


You have learnt in Grade 7 how to classify a triangle according to the lengths of its sides and the magnitudes of its angles.

• Classification of triangles according to the lengths of the sides

Triangle	Figure	Note
Equilateral triangle		The lengths of all three sides are equal
Isosceles triangle		The lengths of two sides are equal
Scalene triangle		All three sides are unequal in length

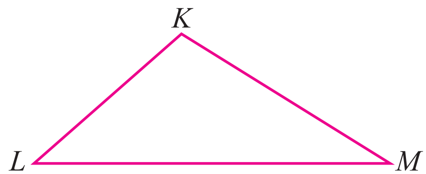
• Classification of triangles according to the angles

Triangle	Figure	Note
Acute triangle		The magnitude of each angle is less than 90º.
Obtuse triangle		The magnitude of one angle is greater than 90º.
Right triangle		The magnitude of one angle is 90º.

Do the following review exercise to recall the facts you learnt in Grade 7 on triangles and angles.

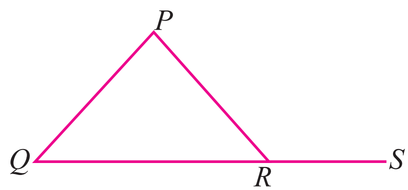
Review Exercise

(1) Name the three sides and the three angles of the triangle shown in the figure.

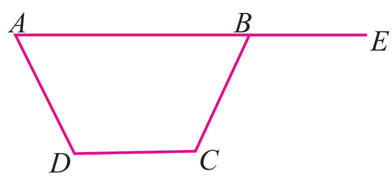


(2) (i) Draw an obtuse triangle and name it ABC.
(ii) Measure $A\hat{B}C$, $B\hat{A}C$ and $A\hat{C}B$ and write down their magnitudes.

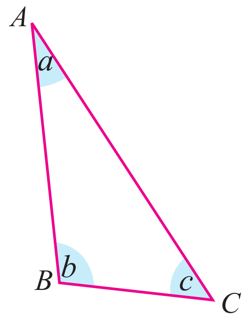
(3) (i) Draw a triangle PQR as in the figure and produce QR to S.
(ii) Measure $P\hat{R}Q$ and $P\hat{R}S$ and write down their magnitudes.



(4) (i) Draw a quadrilateral $ABCD$ and produce AB to E.
(ii) Measure $E\hat{B}C$ and $A\hat{B}C$ and write down their magnitudes.



12.2 The sum of the interior angles of a triangle

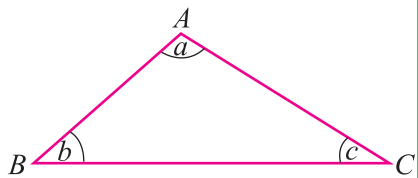


The angles located within the triangle ABC are named a, b, and c. Since they are located within the triangle, they are called the **interior angles of the triangle** ABC**.**

Engage in the following activity in order to find the sum of the interior angles of a triangle.

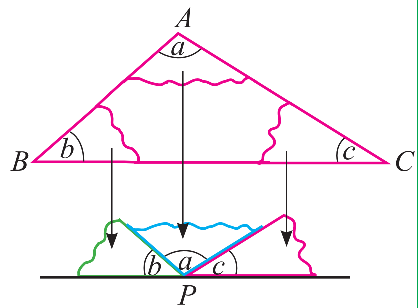
Activity 1

Step 1 - Draw any triangle on a piece of white paper and name its vertices as A,B and C and its interior angles as a,b and c respectively, as shown in the figure.



Step 2 - Cut and separate out the three angles a, b and c as shown in the figure.

Step3 - In your exercise book, paste the three angles a,b and c that were cut out, as shown in the figure, without overlapping them and such that the point P on the line is the common vertex.



Step 4 - Establish the fact that the three pasted angles are located on a straight line, by keeping a ruler. Write down the value of $a + b + c$.

- Draw another triangle in your exercise book, measure the three interior angles and find their sum.

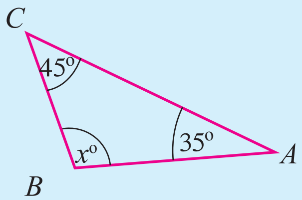
It must be clear to you from the above activity that the sum of the three interior angles of a triangle can be presented as the sum of three angles located on a straight line, completely covering one side of it.

Since the sum of the angles at a point on a straight line is 180°, it can be concluded that the sum of the three interior angles of a triangle is 180°.

The sum of the interior angles of a triangle is 180°.

Example 1

Find the magnitude of $A\hat{B}C$ in the figure.

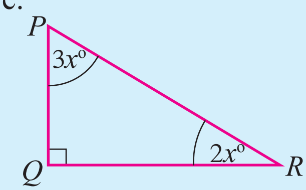


Since the sum of the interior angles of a triangle is 180°,

$$45 + 35 + x = 180$$
$$80 + x = 180$$
$$x + 80 - 80 = 180 - 80$$
$$x = 100$$
$$A\hat{B}C = 100^\circ$$

Example 2

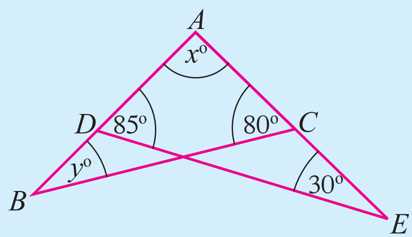
Find the magnitude of $Q\hat{P}R$ in the figure.



$$3x + 2x + 90 = 180$$
$$5x + 90 = 180$$
$$5x + 90 - 90 = 180 - 90$$
$$5x = 90$$
$$\frac{5x}{5} = \frac{90}{5}$$
$$x = 18$$
$$\therefore Q\hat{P}R = 3 \times 18^\circ = 54^\circ$$

Example 3

Find the values of x and y according to the information marked in the figure.



Since the sum of the interior angles of the triangle ADE is 180°,

$$85 + 30 + x = 180$$
$$115 + x = 180$$
$$x + 115 - 115 = 180 - 115$$
$$x = 65$$

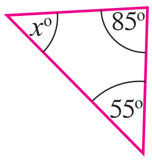
Since the sum of the interior angles of the triangle ABC is 180°,

$$x + 80 + y = 180$$
$$65 + 80 + y = 180 \text{ (substituting } x^\circ = 65^\circ)$$
$$y + 145 = 180$$
$$y + 145 - 145 = 180 - 145$$
$$y = 35$$

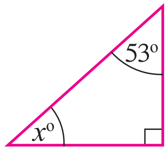
Exercise 12.1

(1) Find the magnitude of the angle marked as x in each figure.

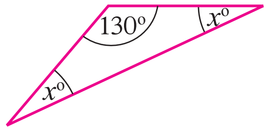
(i)



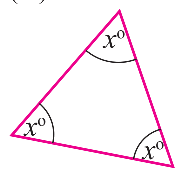
(ii)



(iii)

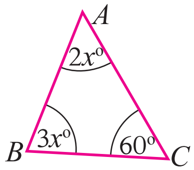


(iv)

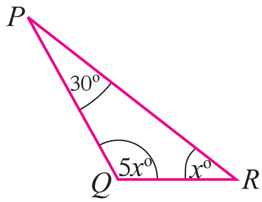


(2) Find the magnitude of each of the angles in each triangle.

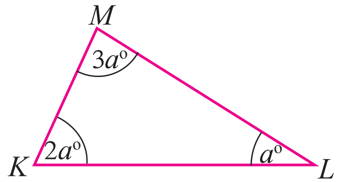
(i)



(ii)

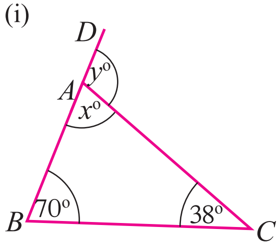


(iii)

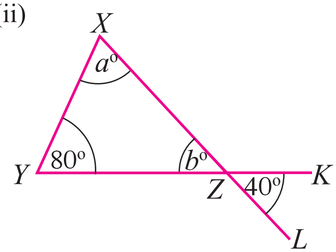


(3) Find the magnitude of each of the angles denoted by an English letter in each figure.

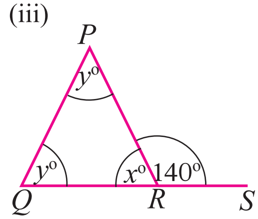
(i)



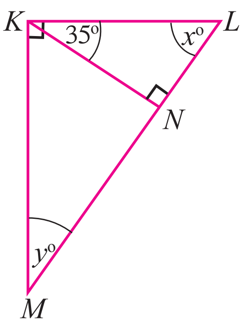
(ii)



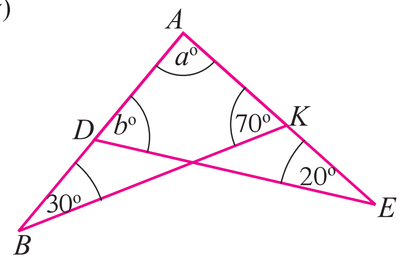
(iii)



(iv)

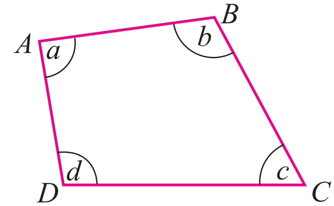


(v)



12.3 The sum of the interior angles of a quadrilateral

You learnt in Grade 6 that a closed rectilinear plane figure which consists of 4 sides is called a **quadrilateral**. A quadrilateral has 4 sides and 4 angles.

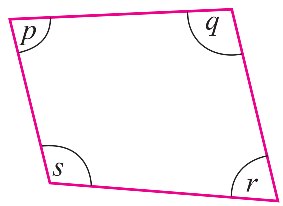


The sides of the quadrilateral *ABCD* are *AB, BC, CD* and *DA*. Its sides can also be named *BA, CB, DC* and *AD*.

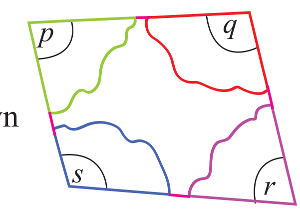
The interior angles of the quadrilateral *ABCD* in the figure are marked as *a*, *b*, *c* and *d*.

Do the activity given below in order to find the sum of the interior angles of a quadrilateral.

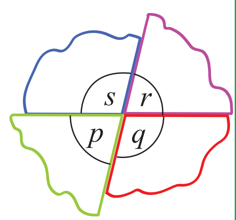
Activity 2



Step 1 - Draw any quadrilateral on a piece of coloured paper and name its interior angles as *p*, *q*, *r* and *s*.



Step 2 - Cut and separate out the angles *p*, *q*, *r*, *s* as shown in the figure.



Step 3 - In your exercise book, paste the angles that were cut out, around a point without overlapping them, such that the vertices of all the angles coincide.

Step 4 - Write down a value for $p+q+r+s$ by considering the sum of the angles around a point.

Step 5 - Draw another quadrilateral in your exercise book, measure its interior angles and obtain a value for their sum.

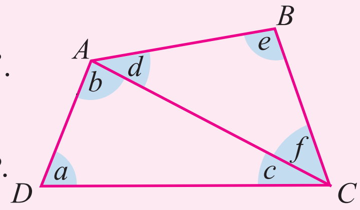
In the above activity, you would have obtained that $p+q+r+s = 360^{\circ}$.

Since the sum of the angles located around a point is 360° , it can be concluded that the sum of the four interior angles of a quadrilateral is 360° .

The sum of the interior angles of a quadrilateral is 360°.

Note:

The quadrilateral *ABCD* is shown in the figure. By joining the vertices *A* and *C*, the triangles *ABC* and *ADC* are created.



The sum of the three angles of the triangle *ADC* is 180°.

Accordingly, $a + b + c = 180^\circ$.

The sum of the three angles of the triangle *ABC* is 180°.

Accordingly, $d + e + f = 180^\circ$.

$\therefore$ The sum of the interior angles of the quadrilateral = The sum of the interior angles of the triangle *ADC* + The sum of the interior angles of the triangle *ABC*

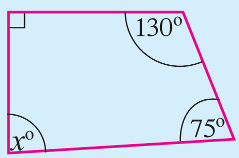
$= (a + b + c) + (d + e + f)$

$= 180^\circ + 180^\circ = 360^\circ$

Accordingly, the sum of the interior angles of a quadrilateral is 360°.

Example 1

Find the value of x in the figure.



Since the sum of the interior angles of a quadrilateral is 360°,

$x + 90 + 130 + 75 = 360$

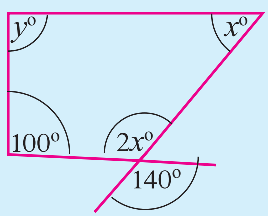
$x + 295 = 360$

$x + 295 - 295 = 360 - 295$

$x = 65$

Example 2

Find the values of x and y in the figure.



Since vertically opposite angles are equal,

$2x = 140$

$x = 70$

Since the sum of the interior angles of a quadrilateral is 360°,

$y + 100 + 2x + x = 360$

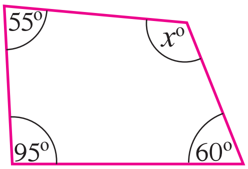
$y + 100 + 140 + 70 = 360$

$y + 310 - 310 = 360 - 310 = 50$

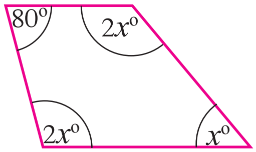
Exercise 12.2

(1) Find the value of x in each figure given below.

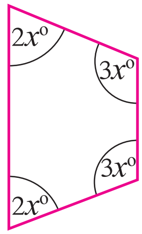
(i)



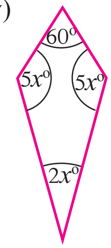
(ii)



(iii)

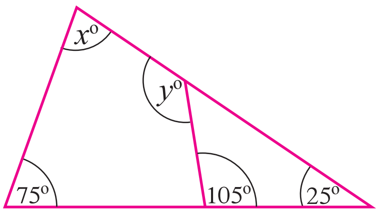


(iv)

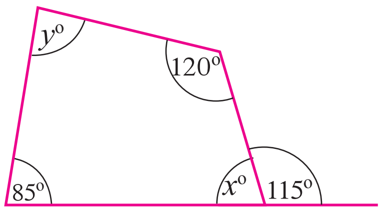


(2) Find the values of x and y in each figure given below.

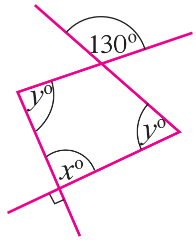
(i)



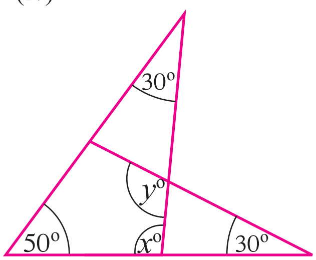
(ii)



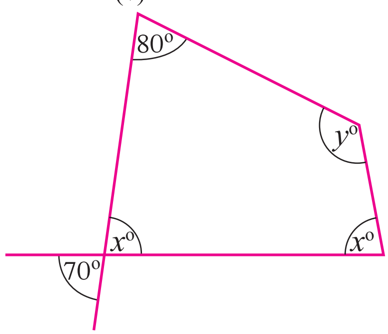
(iii)



(iv)



(v)

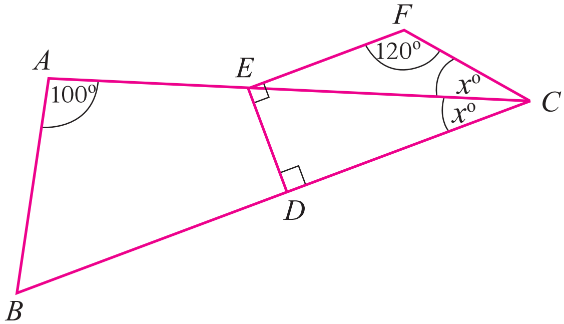


(3) Find the magnitude of each of the following angles, based on the information marked in the figure.

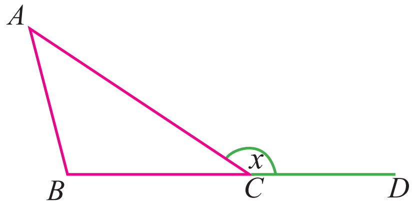
(i) $D\hat{C}F$

(ii) $A\hat{B}C$

(iii) $A\hat{E}D$

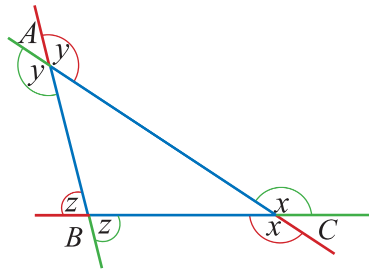


12.4 Exterior angles of a triangle



In the triangle ABC, the side BC is produced to D. The angle ACD with arms AC and CD, coloured in green, which is then formed, is **an exterior angle of the triangle** ABC.

As shown in the figure, more exterior angles can be created by producing the other sides of the triangle ABC.



Although there are two exterior angles formed at every vertex of a triangle, they are equal in magnitude since they are vertically opposite angles.

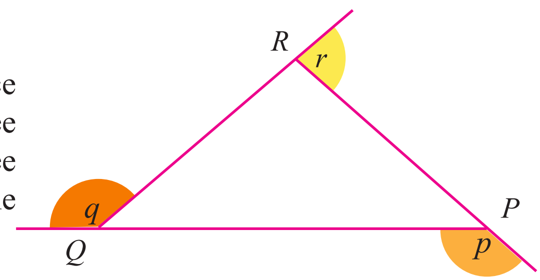
When one exterior angle at each vertex is considered, then the sum of these angles is said to be the **sum of the exterior angles of the triangle**.

• The sum of the exterior angles of a triangle

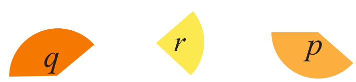
Let us engage in activity 3 in order to obtain a value for the sum of the exterior angles of a triangle.

Activity 3

Step 1 - Draw any triangle on a piece of paper and draw three exterior angles at its three vertices as shown in the figure.

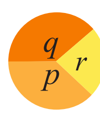


Step 2 - As shown in the figure, cut and separate out laminas of the three exterior angles using a blade.



Step 3 - In your exercise book, paste the three exterior angles (the three laminas) that were cut out, around a point without overlapping them, such that the vertices of the three exterior angles coincide.

Step 4 - Find the sum $p+q+r$ of the exterior angles of the triangle, by using the knowledge on the sum of the angles around a point.



- Draw another triangle in your exercise book, produce the sides to form exterior angles at the three vertices, and by measuring them, obtain the sum of the exterior angles of the triangle.

  It is clear from the above activity, that the three exterior angles of a triangle can be positioned as three angles around a point.

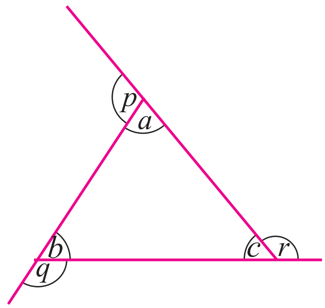
  Since the sum of the angles around a point is 360˚, the sum of the exterior angles of a triangle is also 360˚.

This result is obtained by measuring the angles as well.
According to the given figure,

$$(a+p)+(b+q)+(c+r) = 180^{o}+180^{o}+180^{o}$$
$$= 540^{o}$$
$$\therefore (a+b+c)+(p+q+r) = 540^{o}$$

Since, $a+b+c = 180^{o}$,

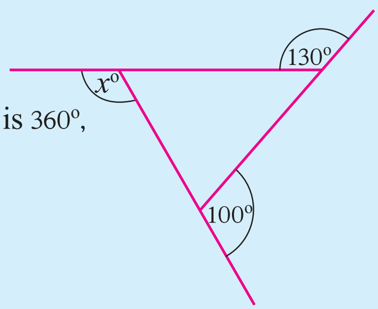
$$180^{o}+(p+q+r) = 540^{o}$$
$$\therefore p+q+r = 540^{o}-180^{o}$$
$$= 360^{o}$$



The sum of the exterior angles of a triangle is 360˚.

Example 1

Find the value of x in the figure.

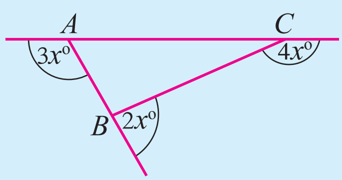


Since the sum of the exterior angles of a triangle is 360^{o},

$$130+100+x = 360$$
$$230+x = 360$$
$$x+230-230 = 360-230$$
$$x = 130$$

Example 2

Find the magnitudes of the three exterior and the three interior angles of the triangle ABC.



$$3x + 2x + 4x = 360$$

$$9x = 360$$

$$\frac{9x}{9} = \frac{360}{9}$$

$$\therefore x = 40$$

$\therefore$ The exterior angle at vertex $A = 3x^{o} = 3 \times 40^{o} = 120^{o}$

The exterior angle at vertex $B = 2x^{o} = 2 \times 40^{o} = 80^{o}$

The exterior angle at vertex $C = 4x^{o} = 4 \times 40^{o} = 160^{o}$

Since the sum of the angles on a straight line is 180˚,

the interior angle at vertex $A = 180^{o} - 120^{o} = 60^{o}$

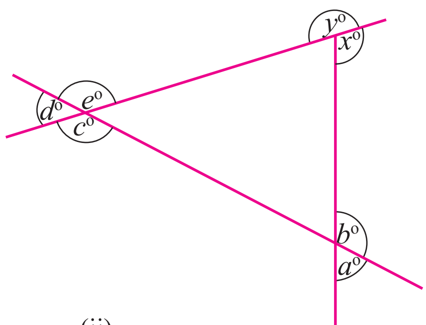
the interior angle at vertex $B = 180^{o} - 80^{o} = 100^{o}$

the interior angle at vertex $C = 180^{o} - 160^{o} = 20^{o}$

Exercise 12.3

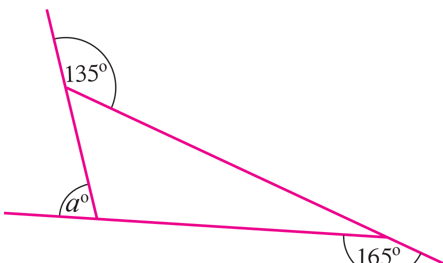
(1) (i) Select and write the exterior angles from among the angles a, b, c, d, e, x and y shown in the figure.

(ii) Explain why the other angles are not exterior angles.

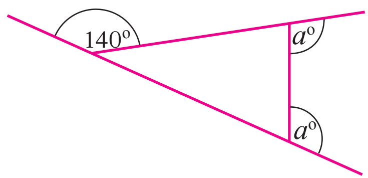


(2) Find the value of each of the angles denoted by an English letter in each figure given below.

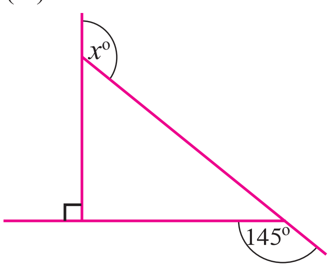
(i)



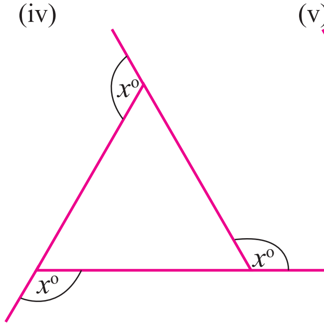
(ii)



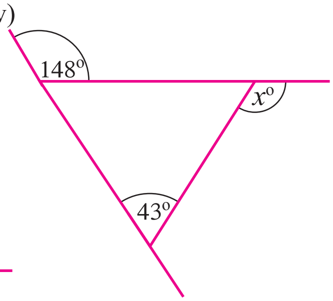
(iii)



(iv)



(v)

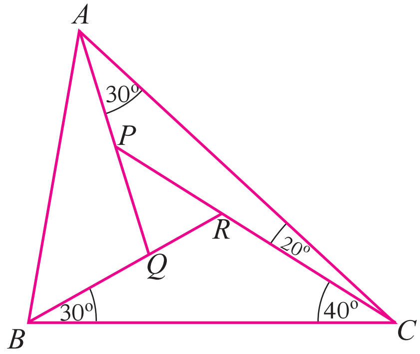


(3) According to the information marked in the figure,

(i) find $B\hat{R}C$.

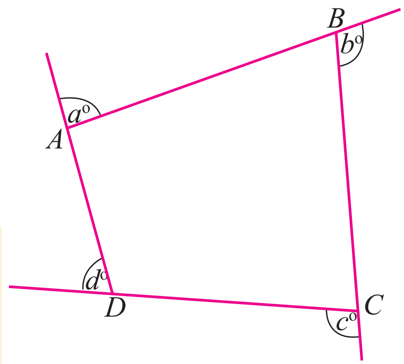
(ii) find $A\hat{P}C$.

(iii) find $B\hat{Q}A$.



12.5 Exterior angles of a quadrilateral

The exterior angles created by producing the sides of the quadrilateral *ABCD* are marked in the figure as *a*, *b*, *c* and *d* .



A quadrilateral has four vertices. Hence, there are four exterior angles.

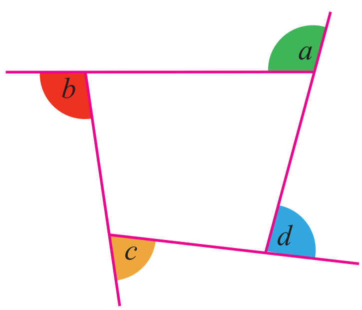
Although there are two exterior angles formed at every vertex of a quadrilateral, they are equal in magnitude since they are vertically opposite angles.

When one exterior angle at each vertex is considered, then the sum of these angles is said to be the **sum of the exterior angles of the quadrilateral**.

Let us engage in the activity given below in order to find the sum of the exterior angles of a quadrilateral.

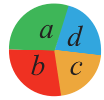
Activity 4

Step 1 - Draw any quadrilateral, and draw four exterior angles at its four vertices.



Step 2 - As shown in the figure, cut and separate out laminas of the exterior angles with a blade.

Step 3 - Obtain a value for $a + b + c + d$, by pasting the four exterior angles that were cut out, around a point without overlapping them, such that their vertices coincide.



➢ Draw another quadrilateral in your exercise book and obtain a value for the sum of its exterior angles by measuring them.

It is clear from the above activity that the sum of the exterior angles of a quadrilateral is 360˚.

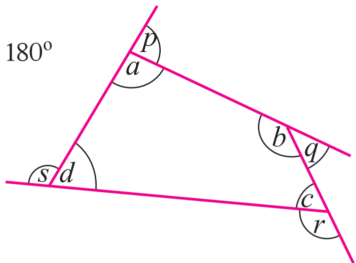
The sum of the exterior angles of a quadrilateral is 360˚.

According to the given figure,

$$a + p + b + q + c + r + d + s = 180^\circ + 180^\circ + 180^\circ + 180^\circ$$
$$(a + b + c + d) + (p + q + r + s) = 720^\circ$$

Since $a + b + c + d = 360^\circ$,

$$360^\circ + p + q + r + s = 720^\circ$$
$$= 720^\circ - 360^\circ$$
$$= 360^\circ$$

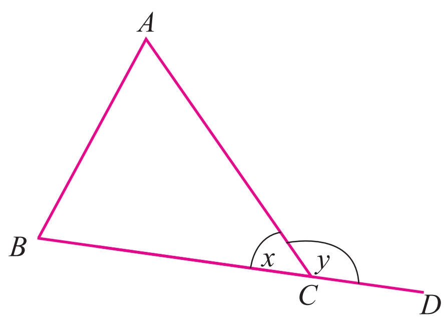


• The sum of an exterior angle and an interior angle at one vertex of a triangle and of a quadrilateral

The interior and exterior angles of a triangle at one vertex are shown in the figure as x and y.

These two angles are located on the straight line BD, at the point C.

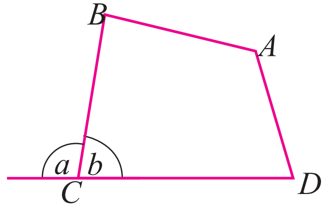
Since the sum of the angles at a point on a straight line is 180˚, $x + y = 180^\circ$.



At a vertex of a triangle, interior angle + exterior angle $= 180^\circ$.

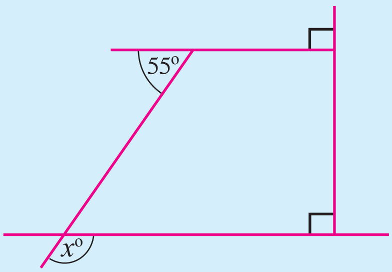
As for a triangle, the sum of the interior angle and the exterior angle at each vertex of a quadrilateral is 180°.

$\therefore \quad a + b = 180^\circ$



Example 1

Find the value of x.



$x + 55 + 90 + 90 = 360$

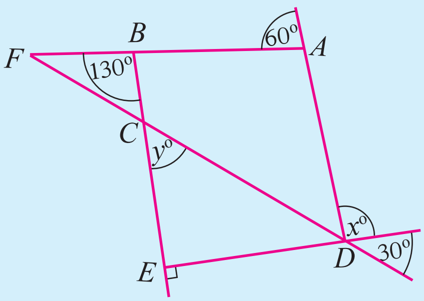
$x + 235 = 360$

$x = 360 - 235$

$x = 125$

Example 2

Find the values of x and y according to the information marked in the figure.



Since the sum of the exterior angles of the quadrilateral $ABED$ is 360°,

$60 + 130 + 90 + x = 360$

$x + 280 = 360$

$x + 280 - 280 = 360 - 280$

$x = 80$

By taking the sum of the exterior angles of the quadrilateral $ABCD$,

$60 + 130 + y + (30 + x) = 360$

$190 + y + 30 + 80 = 360$

$y + 300 = 360$

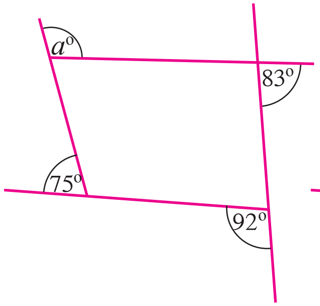
$y = 360 - 300$

$y = 60$

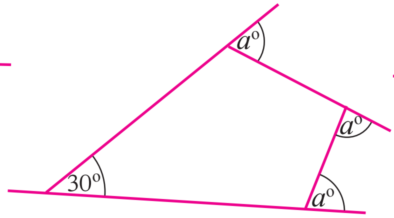
Exercise 12.4

(1) Find the value of a, marked in each figure.

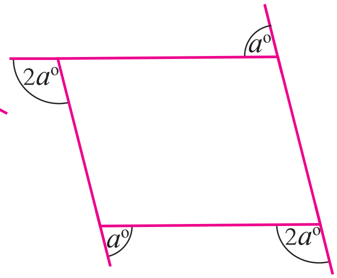
(i)



(ii)

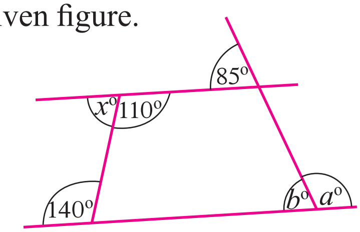


(iii)



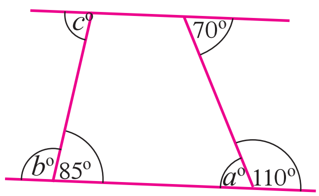
(2) Find the value of each of the angles based on the given figure.

(i) What is the value of x?
(ii) What is the value of a?
(iii) What is the value of b?

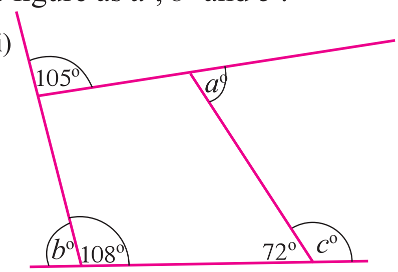


(3) Find the magnitudes of the angles marked in the figure as a°, b° and c°.

(i)

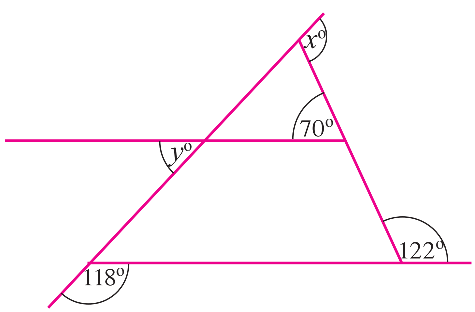


(ii)

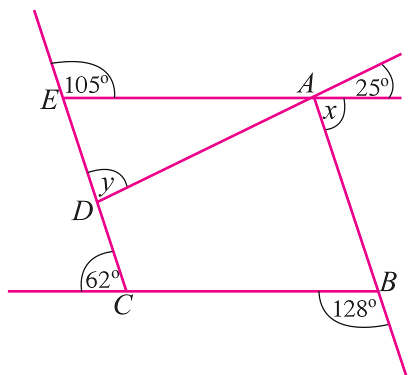


(4) Find the values of x and y in each figure.

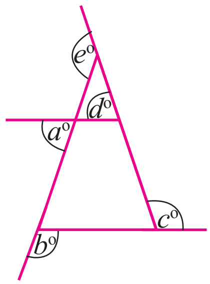
(i)



(ii)



(5)



(i) What is the value of $a + b + c + d$?

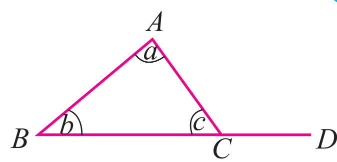
(ii) What is the value of $b + c + e$?

(iii) According to the answers of (i) and (ii), show that $e = a + d$.

Summary

- The sum of the interior angles of a triangle is 180°.
- The sum of the interior angles of a quadrilateral is 360°.
- The sum of the exterior angles of a triangle is 360°.
- The sum of the exterior angles of a quadrilateral is 360°.

Think

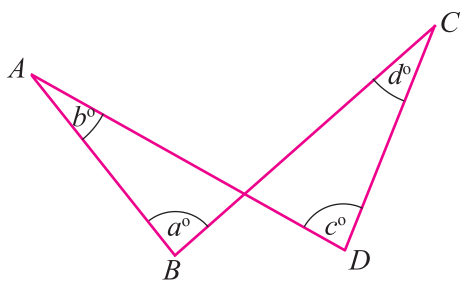


(1) Show that $A\hat{C}D = a + b$.

(2) (i) $ABCD$ is not a polygon. Explain the reason.

(ii) $a + b = c + d$. Explain the reason.

(iii) Show that the value of $a + b + c + d$ is less than 360°.



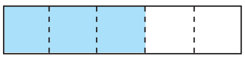
13 Fractions I

By studying this lesson, you will be able to,

- multiply a fraction by a whole number,
- multiply a fraction by a fraction,
- multiply a fraction by a mixed number, and
- multiply a mixed number by a mixed number.

13.1 Fractions

Let us first recall what you have learnt about fractions in Grades 6 and 7. Let us take the area of the figure given below as a unit.



This unit has been divided into five equal parts of which three parts are coloured. You have learnt that the coloured area is $\frac{3}{5}$ of the whole area.

You have also learnt that if a unit is divided into equal parts, then one part or several of these parts is called a fraction of the unit. A portion of a collection is also considered as a fraction of that collection.

In addition, you have learnt that fractions such as $\frac{3}{5}$, $\frac{1}{2}$ and $\frac{2}{3}$ which are less than one and greater than zero are called proper fractions.

A number which has been written by adding together a whole number and a proper fraction is called a **mixed number** or an **improper fraction**, depending on how it has been represented.

Some examples of mixed numbers are $1\frac{1}{2}$, $2\frac{1}{3}$ and $4\frac{2}{5}$.

In the mixed number $4\frac{2}{5}$, the whole part is 4 and the fractional part is $\frac{2}{5}$.

Some examples of improper fractions are $\frac{3}{2}$, $\frac{5}{3}$ and $\frac{11}{7}$.

The numerator of an improper fraction is greater than or equal to the denominator.

A fraction **equivalent** to a given fraction can be obtained by multiplying the numerator and the denominator of the fraction by the same non-zero number.

A fraction **equivalent** to a given fraction can also be obtained by dividing the numerator and the denominator by a non - zero common factor of the numerator and the denominator.

- **Representing a mixed number as an improper fraction**

By following the steps given below, a mixed number can be represented as an improper fraction.

- Multiply the whole number part of the mixed number by the denominator of the fractional part, and add it to the numerator of the fractional part.
- The denominator of the improper fraction is the same as the denominator of the fractional part of the mixed number.

- **Representing an improper fraction as a mixed number**

You learnt in Grade 7 how to represent an improper fraction as a mixed number.

Let us represent $\frac{7}{4}$ as a mixed number.

Method I

$$\frac{7}{4} = \frac{4+3}{4}$$

$$= \frac{4}{4} + \frac{3}{4}$$

$$= 1 + \frac{3}{4} = 1\frac{3}{4}$$

Method II

$$\frac{7}{4} = 7 \div 4$$

$$\begin{array}{r|l} & 1 \\ 4 & 7 \\ & 4 \\ \hline & 3 \end{array}$$

The quotient and remainder of 7 ÷ 4 are 1 and 3 respectively. The quotient is the whole number part of the mixed number and the remainder is the numerator of the fractional part.

The denominator of the fractional part of the mixed number is the same as the denominator of the improper fraction.

$$\therefore \frac{7}{4} = 1\frac{3}{4}$$

You have learnt how to add and subtract fractions in Grades 6 and 7.

Do the following review exercise to recall what you have learnt previously about fractions.

Review Exercise

(1) Choose the appropriate value from the brackets and fill in the blanks.

(i) $\frac{3}{4}$ is .... $\frac{1}{4}$ths (two, three, five)

(ii) $\frac{2}{5}$ is two .... $\left(\frac{1}{3}\text{rds}, \frac{1}{2}\text{s}, \frac{1}{5}\text{ths}\right)$

(iii) Four $\frac{1}{7}$ths is ...... $\left(\frac{4}{7}, \frac{4}{5}, \frac{4}{9}\right)$

(2) Write down two equivalent fractions for each fraction given below.

(i) $\frac{3}{4}$ (ii) $\frac{2}{5}$ (iii) $\frac{6}{10}$ (iv) $\frac{8}{24}$

(3) Represent each mixed number given below as an improper fraction.

(i) $1\frac{1}{5}$ (ii) $3\frac{3}{5}$ (iii) $6\frac{1}{6}$

(4) Represent each improper fraction given below as a mixed number.

(i) $\frac{14}{5}$ (ii) $\frac{18}{7}$ (iii) $\frac{37}{3}$

(5) Simplify the following.

(i) $\frac{2}{5} + \frac{1}{5}$ (ii) $\frac{1}{3} + \frac{1}{2}$ (iii) $\frac{3}{5} + \frac{1}{3}$ (iv) $\frac{7}{12} + \frac{1}{8}$

(v) $\frac{1}{6} + \frac{5}{8}$ (vi) $\frac{11}{15} + \frac{2}{10}$ (vii) $1\frac{1}{2} + 4\frac{3}{8}$ (viii) $2\frac{1}{4} + 3\frac{5}{9}$

(6) Simplify the following.

(i) $\frac{6}{7} - \frac{2}{7}$ (ii) $\frac{7}{10} - \frac{2}{5}$ (iii) $\frac{1}{3} - \frac{2}{7}$ (iv) $1 - \frac{1}{5}$

(v) $\frac{7}{8} - \frac{5}{6}$ (vi) $3\frac{7}{8} - 1\frac{1}{2}$ (vii) $3 - 1\frac{5}{8}$ (viii) $2\frac{2}{5} - 1\frac{3}{20}$

13.2 Multiplying a fraction by a whole number

The figure depicts a cake, which is divided into five equal parts.

We know that one part of the entire cake is $\frac{1}{5}$ of the cake.
Let us take 3 such parts.

$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$

Let us consider how much the total of these three parts is from the entire cake. For this, we have to add these three quantities.

It is $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{3}{5}$

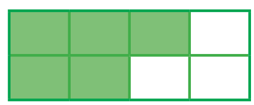
You have learnt previously that addition of the same number repeatedly can be represented as a multiplication.

For example, $2+2+2=2\times 3=6$

Accordingly, we can write $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{1}{5}\times 3$

Therefore, $\frac{1}{5}\times 3=\frac{3}{5}$. That is, three $\frac{1}{5}$ is equal to $\frac{3}{5}$.

- The figure depicts a rectangle which has been divided into eight equal parts. One part is $\frac{1}{8}$ of the entire figure.



Let us consider the sum of 5 such parts.
It can be written as $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{5}{8}$

That is, five $\frac{1}{8}$ s is equal to $\frac{5}{8}$

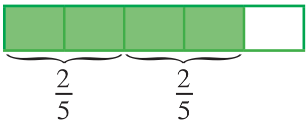
$\frac{1}{8}\times 5=\frac{5}{8}$.

Accordingly,

$\frac{1}{3} \times 1 = \frac{1}{3}$, $\frac{1}{3} \times 2 = \frac{2}{3}$, $\frac{1}{10} \times 7 = \frac{7}{10}$

- Now let us consider a multiplication of the form $\frac{2}{5} \times 2$.

Let us represent this by a figure.



This can be written as $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$

When this sum is written as a product we obtain,

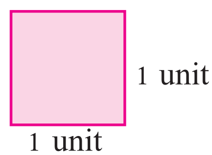
$\frac{2}{5} \times 2 = \frac{4}{5}$

When a fraction is multiplied by a whole number, the numerator of the resultant fraction is the product of the whole number and the numerator of the given fraction, and its denominator is the same as that of the given fraction.

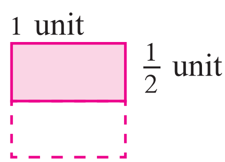
• Multiplying a whole number by a fraction

You have learnt that the area of a square shaped lamina of length 1 unit and breadth 1 unit is 1 square unit.

That is, the area of the square shaped lamina = 1 unit × 1 unit
= 1 square unit



Now let us find the area of a rectangular shaped lamina which is of length 1 unit and breadth $\frac{1}{2}$ a unit using two methods.



Method I

Since the area of this rectangular shaped lamina is $\frac{1}{2}$ the area of the square of area 1 square unit, the area of the rectangular lamina is $\frac{1}{2}$ square units.

Method II

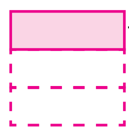
Since the length of this lamina is 1 unit and its breadth is $\frac{1}{2}$ a unit,

area of the lamina $=$ (length $\times$ breadth) square units

$= 1 \times \frac{1}{2}$ square units

$$\therefore\ 1 \times \frac{1}{2} = \frac{1}{2}$$

Furthermore, the area of the rectangular shaped lamina in the figure which is of length 1 unit and breadth $\frac{1}{3}$ units is $\frac{1}{3}$ square units.



That is, $1 \times \frac{1}{3} = \frac{1}{3}$

You have learnt in the previous section that $\frac{1}{3} \times 1 = \frac{1}{3}$

$$\therefore\ \frac{1}{3} \times 1 = 1 \times \frac{1}{3}$$

Similarly,

$\frac{2}{7} \times 3 = \frac{6}{7}$ and $3 \times \frac{2}{7} = \frac{6}{7}$

$\frac{4}{11} \times 2 = \frac{8}{11}$ and $2 \times \frac{4}{11} = \frac{8}{11}$

$\frac{2}{13} \times 5 = \frac{10}{13}$ and $5 \times \frac{2}{13} = \frac{10}{13}$

$\therefore\ \frac{2}{7} \times 3 = 3 \times \frac{2}{7}$

$\therefore\ \frac{4}{11} \times 2 = 2 \times \frac{4}{11}$

$\therefore\ \frac{2}{13} \times 5 = 5 \times \frac{2}{13}$

When multiplying a fraction by a whole number, and when multiplying the same whole number by the same fraction we obtain the same answer.

Example 1

(i) Simplify $\frac{3}{7} \times 2$.

$\frac{3}{7} \times 2 = \frac{3 \times 2}{7}$

$= \frac{6}{7}$

Example 2

(ii) Simplify $\frac{3}{8} \times 5$.

$\frac{3}{8} \times 5 = \frac{3 \times 5}{8}$

$= \frac{15}{8}$

$= 1\frac{7}{8}$

Example 3

(iii) Simplify $4 \times \frac{2}{5}$.

$4 \times \frac{2}{5} = \frac{4 \times 2}{5}$

$= \frac{8}{5}$

$= 1\frac{3}{5}$

Exercise 13.1

(1) Express the product of each of the following in its simplest form (If the answer is an improper fraction, express it as a mixed number).

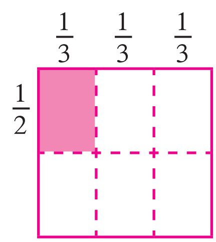
(i) $\frac{1}{6} \times 5$	(ii) $\frac{3}{10} \times 3$	(iii) $6 \times \frac{2}{13}$	(iv) $\frac{3}{7} \times 5$
(v) $\frac{2}{7} \times 9$	(vi) $\frac{1}{10} \times 17$	(vii) $5 \times \frac{7}{9}$	(viii) $\frac{3}{4} \times 12$
(ix) $\frac{2}{5} \times 10$	(x) $\frac{7}{8} \times 1$	(xi) $\frac{2}{3} \times 0$	(xii) $4 \times \frac{3}{5}$
(xiii) $3 \times \frac{1}{4}$	(xiv) $\frac{5}{6} \times 8$	(xv) $10 \times \frac{3}{5}$	

(2) A vehicle that travels at a constant speed, journeys $\frac{3}{4}$ kilometers in a minute. How far does it travel in 8 minutes?

(3) A machine produces 600 plastic cups in an hour. How many cups does it produce in $\frac{2}{3}$ hours?

13.3 Multiplying a fraction by a fraction

The figure shows a square shaped lamina of side length 1 unit. It is divided into 6 equal parts, of which one part is shaded as in the figure.



Since the shaded part is $\frac{1}{6}$ of the whole area of the lamina, its area is $\frac{1}{6}$ square units.

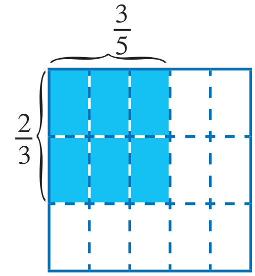
Also, the shape of the shaded part is rectangular. Its length is $\frac{1}{2}$ the length of the square lamina and its breadth is $\frac{1}{3}$ the breadth of the square lamina.

The area of the rectangular shaped lamina is calculated by multiplying its length by its breadth.

Therefore, the area of the shaded part can be written as $\frac{1}{2} \times \frac{1}{3}$ square units. Since this is equal to $\frac{1}{6}$ square units,

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

The figure shows a square shaped lamina of side length 1 unit. It is divided into 15 equal parts.



Let us find the area of the shaded part using two different methods.

Method I

Since the area of the shaded part is $\frac{6}{15}$ of the area of the whole lamina, the area of this part is $\frac{6}{15}$ square units.

Method II

The length of the shaded part of the rectangular shape = $\frac{2}{3}$ of the length of the square (that is, $\frac{2}{3}$ units)

Its breadth = $\frac{3}{5}$ of the length of the square (that is, $\frac{3}{5}$ units).

$\therefore$ The area of the shaded part is $\frac{3}{5} \times \frac{2}{3}$ square units.

$$\therefore \frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$$

Let us consider the above two cases.

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \quad \left(\frac{1 \times 1}{2 \times 3} = \frac{1}{6}\right)$$

$$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} \quad \left(\frac{3 \times 2}{5 \times 3} = \frac{6}{15}\right)$$

When two fractions are multiplied,

- the numerator of the resultant fraction is the product of the two numerators.
- the denominator of the resultant fraction is the product of the two denominators.

Note

- When any fraction is multiplied by zero, the result is zero.

$$\frac{1}{2} \times 0 = \frac{1}{2} \times \frac{0}{1} = \frac{1 \times 0}{2 \times 1} = \frac{0}{2} = 0$$

- When any fraction is multiplied by 1, the result is the same fraction.

$$\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{1}{1} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}$$

Example 1

Simplify

(i) $\frac{4}{7} \times \frac{2}{3}$

$$\frac{4}{7} \times \frac{2}{3} = \frac{4 \times 2}{7 \times 3} = \frac{8}{21}$$

(ii) $\frac{3}{8} \times \frac{4}{5} \times \frac{1}{2}$

$$\frac{3}{8} \times \frac{4}{5} \times \frac{1}{2} = \frac{3 \times 4 \times 1}{8 \times 5 \times 2} = \frac{12}{80}$$

$$= \frac{12 \div 4}{80 \div 4} \text{ (equivalent fraction)}$$

$$= \frac{3}{20}$$

Note

$$\frac{3}{8} \times \frac{4}{5} = \frac{12}{40}$$

In the fraction $\frac{12}{40}$, since 4 is a common factor of both the numerator and the denominator, let us divide the numerator as well as the denominator by 4.

$$\frac{12}{40} = \frac{12 \div 4}{40 \div 4} = \frac{3}{10}$$

This is written as $\frac{\cancel{12}^{3}}{\cancel{40}_{10}} = \frac{3}{10}$.

$$\frac{3}{8} \times \frac{4}{5} = \frac{\cancel{12}^{3}}{\cancel{40}_{10}} = \frac{3}{10}.$$

Also,

$$\frac{3}{8} \times \frac{4}{5} = \frac{3 \times 4}{8 \times 5} = \frac{3 \times 4}{2 \times 4 \times 5}$$

Now, since 4 is the greatest common factor of the numerator and the denominator, by dividing the numerator and the denominator by 4 we obtain,

$$\frac{3 \times \cancel{4}^{1}}{2 \times \cancel{4}_{1} \times 5} = \frac{3}{10}$$

When simplifying $\frac{3}{8} \times \frac{4}{5}$, it is easy to first divide the numerator and the denominator by their common factors.

$$\frac{3}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{5} = \frac{3 \times 1}{2 \times 5} = \frac{3}{10}$$

Exercise 13.2

(1) Simplify the following.

(a) (i) $\frac{1}{2} \times \frac{1}{4}$ (ii) $\frac{2}{3} \times \frac{1}{5}$ (iii) $\frac{3}{4} \times \frac{5}{7}$ (iv) $\frac{3}{5} \times \frac{2}{7}$

(v) $\frac{3}{8} \times \frac{2}{5}$ (vi) $\frac{7}{10} \times \frac{3}{14}$ (vii) $\frac{5}{12} \times \frac{4}{7}$ (viii) $\frac{6}{7} \times \frac{14}{15}$

(b) (i) $\frac{6}{7} \times \frac{3}{8}$ (ii) $\frac{3}{5} \times \frac{2}{3}$ (iii) $\frac{2}{11} \times \frac{3}{4}$ (iv) $\frac{3}{10} \times \frac{5}{6}$

(v) $\frac{3}{4} \times \frac{2}{3}$ (vi) $\frac{5}{12} \times \frac{3}{10}$ (vii) $\frac{1}{2} \times \frac{1}{4} \times \frac{3}{5}$ (viii) $\frac{2}{3} \times \frac{5}{8} \times \frac{3}{10}$

13.4 Multiplying a fraction by a mixed number

Let us now consider how to multiply a fraction by a mixed number.

Let us multiply $\frac{3}{5}$ by $1\frac{1}{2}$.

That is, let us find the value of $\frac{3}{5} \times 1\frac{1}{2}$.

Let us first represent the mixed number as an improper fraction.

$$\frac{3}{5} \times 1\frac{1}{2} = \frac{3}{5} \times \frac{3}{2}$$
$$= \frac{3 \times 3}{5 \times 2}$$
$$= \frac{9}{10}$$

When simplifying fractions which include mixed numbers, multiplication is made easier by first converting the mixed numbers into improper fractions.

Example 1

Simplify $\frac{2}{3} \times 1\frac{1}{4}$.

$\frac{2}{3} \times 1\frac{1}{4} = \frac{{}^{1}\cancel{2}}{3} \times \frac{5}{\cancel{4}_{2}}$ (divide 2 and 4 by 2)

$$= \frac{1 \times 5}{3 \times 2}$$
$$= \frac{5}{6}$$

Example 2

Simplify $1\frac{3}{5} \times \frac{3}{4}$.

$1\frac{3}{5} \times \frac{3}{4} = \frac{{}^{2}\cancel{8}}{5} \times \frac{3}{\cancel{4}_{1}}$ (divide 4 and 8 by 4)

$$= \frac{2 \times 3}{5 \times 1}$$
$$= \frac{6}{5}$$
$$= 1\frac{1}{5}$$

Exercise 13.3

(1) Simplify the following.

(i) $\frac{2}{3} \times 1\frac{1}{3}$ (ii) $\frac{3}{5} \times 1\frac{1}{4}$ (iii) $\frac{5}{8} \times 1\frac{2}{3}$ (iv) $\frac{7}{10} \times 2\frac{1}{7}$

(v) $\frac{1}{6} \times 2\frac{1}{5}$ (vi) $\frac{3}{5} \times 3\frac{1}{9}$ (vii) $\frac{7}{10} \times 33\frac{1}{3}$ (viii) $\frac{5}{12} \times 3\frac{3}{11}$

(ix) $2\frac{1}{2} \times \frac{1}{5}$ (x) $3\frac{3}{4} \times \frac{7}{10}$ (xi) $\frac{2}{5} \times \frac{1}{2} \times 2\frac{1}{2}$ (xii) $\frac{3}{4} \times \frac{2}{5} \times 1\frac{1}{6}$

(2) If a vehicle travels a distance of $12\frac{1}{2}$ km on 1 l of fuel, find the distance it travels on $\frac{3}{4}$ l of fuel.

(3) Aheli reads a certain book for $1\frac{3}{4}$ hours each day. She finishes reading the book in 7 days. Find in hours, the time she took to finish the book.

(4) When kamala was hospitalized due to an illness, the doctor instructed her to drink $\frac{1}{10}$ l of liquid once every $\frac{1}{2}$ hour. Calculate the amount of liquid that kamala drinks during $3\frac{1}{2}$ hours in millilitres.

13.5 Multiplying a mixed number by a mixed number

When multiplying a mixed number by a mixed number, first write each mixed number as an improper fraction.

Let us simplify $1\frac{1}{2} \times 1\frac{2}{5}$.

$1\frac{1}{2} \times 1\frac{2}{5} = \frac{3}{2} \times \frac{7}{5}$ (first the mixed numbers need to be written as improper fractions)

$= \frac{3 \times 7}{2 \times 5}$

$= \frac{21}{10} = 2\frac{1}{10}$

Example 1

Simplify $1\frac{3}{5} \times 2\frac{3}{4}$.

$$1\frac{3}{5} \times 2\frac{3}{4} = \frac{\cancel{8}^{2}}{5} \times \frac{11}{\cancel{4}_{1}}$$

$$= \frac{2 \times 11}{5 \times 1}$$

$$= \frac{22}{5} = 4\frac{2}{5}$$

Example 2

Simplify $1\frac{1}{4} \times 3\frac{1}{2} \times \frac{1}{4}$.

$$1\frac{1}{4} \times 3\frac{1}{2} \times \frac{1}{4} = \frac{5}{4} \times \frac{7}{2} \times \frac{1}{4}$$

$$= \frac{35}{32}$$

$$= 1\frac{3}{32}$$

Exercise 13.4

(1) Simplify the following.

(i) $2\frac{1}{2} \times 1\frac{3}{5}$ (ii) $1\frac{1}{2} \times 4\frac{1}{3}$ (iii) $3\frac{3}{4} \times 1\frac{1}{5}$ (iv) $1\frac{2}{3} \times 3\frac{3}{4}$

(v) $6\frac{1}{4} \times 2\frac{2}{5}$ (vi) $10\frac{2}{3} \times 2\frac{1}{4}$ (vii) $1\frac{3}{7} \times 1\frac{1}{100}$ (viii) $5\frac{1}{4} \times 2\frac{2}{7}$

(ix) $3\frac{1}{2} \times 4\frac{4}{5} \times \frac{5}{14}$ (x) $3\frac{3}{10} \times 2\frac{1}{3} \times 4\frac{2}{7}$

Summary

- When a fraction is multiplied by a whole number, the numerator of the resultant fraction is the product of the whole number and the numerator of the given fraction, and its denominator is the same as that of the given fraction
- When a fraction is multiplied by a fraction, the numerator of the resultant fraction is the product of the numerators of the given fractions and its denominator is the product of the denominators of the given fractions.

14 Fractions II

By studying this lesson, you will be able to,

- write the reciprocal of a whole number and of a fraction,
- divide a fraction by a whole number and a whole number by a fraction,
- divide a fraction by a fraction,
- divide a whole number by a mixed number,
- divide a mixed number by a whole number,
- divide a fraction by a mixed number, and a mixed number by a fraction, and
- divide a mixed number by a mixed number.

14.1 Reciprocal of a number

Using the previously gained knowledge on multiplying fractions, let us now examine the following.

$$2 \times \frac{1}{2} = \frac{2}{2} = 1$$

$$\frac{1}{3} \times 3 = \frac{3}{3} = 1$$

$$7 \times \frac{1}{7} = \frac{7}{7} = 1$$

$$\frac{2}{5} \times \frac{5}{2} = \frac{10}{10} = 1$$

$$\frac{3}{8} \times \frac{8}{3} = \frac{24}{24} = 1$$

In each of the cases shown above, the product of the two fractions is 1.

As in the above cases, if the product of two numbers is 1, then each is called the **reciprocal** of the other.

Accordingly,

since $2 \times \frac{1}{2} = 1$,

$\frac{1}{2}$ is the reciprocal of 2. Also, 2 is the reciprocal of $\frac{1}{2}$.

Also, since $3 \times \frac{1}{3} = 1$,

$\frac{1}{3}$ is the reciprocal of 3 and 3 is the reciprocal of $\frac{1}{3}$.

Furthermore, since $\frac{2}{5} \times \frac{5}{2} = 1$,

$\frac{2}{5}$ is the reciprocal of $\frac{5}{2}$ and $\frac{5}{2}$ is the reciprocal of $\frac{2}{5}$.

Note

A whole number can also be written as a fraction, taking the whole number as the numerator and 1 as the denominator as in $3 = \frac{3}{1}$.

Number	Reciprocal
2	$\frac{1}{2}$
$\frac{1}{3}$	3
$\frac{2}{5}$	$\frac{5}{2}$
$\frac{3}{8}$	$\frac{8}{3}$

- The numerator of the reciprocal of a fraction is the denominator of that fraction, while its denominator is the numerator of that fraction.
- It is clear that the reciprocal of a fraction is obtained by interchanging its numerator and its denominator.

• Reciprocal of a mixed number

When finding the reciprocal of a mixed number such as $1\frac{1}{2}$, first the mixed number is written as an improper fraction.

Accordingly, $1\frac{1}{2} = \frac{3}{2}$

Since the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$, the reciprocal of $1\frac{1}{2}$ is $\frac{2}{3}$.

Note:

Since there is no number which when multiplied by 0 (Zero) gives 1, 0 has no reciprocal.

Exercise 14.1

(1) Fill in the blanks using the correct values.

(i) $\frac{3}{4} \times \frac{\square}{3} = 1$ (ii) $\frac{5}{8} \times \frac{8}{\square} = 1$ (iii) $7 \times \frac{\square}{7} = 1$

(iv) $\frac{1}{5} \times \square = 1$ (v) $1\frac{1}{3} \times \frac{3}{4} = \frac{\square}{3} \times \frac{3}{4} = 1$ (vi) $2\frac{1}{2} \times \frac{2}{\square} = \frac{\square}{2} \times \frac{2}{\square} = 1$

(2) Write down the reciprocal of each of the following numbers.

(i) 6 (ii) $\frac{1}{9}$ (iii) $\frac{5}{7}$ (iv) $\frac{8}{3}$

(v) 1 (vi) $3\frac{1}{3}$ (vii) $2\frac{3}{5}$ (viii) $1\frac{5}{9}$

14.2 Dividing a fraction by a whole number

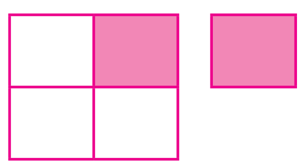
The picture shows a whole cake and $\frac{1}{2}$ a cake.

Suppose we want to share this portion ($\frac{1}{2}$ a cake) equally between Kamal and Amal. Let us consider the share that one person gets from the whole cake, when half the cake is divided equally between them.

This share is $\frac{1}{2} \div 2$.

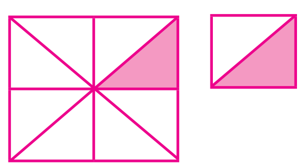
It is clear from the picture that this share is $\frac{1}{4}$ of the whole cake.

Accordingly, $\frac{1}{2} \div 2 = \frac{1}{4}$



The figure on the right hand side shows a square shaped card of which $\frac{1}{4}$ has been coloured.

If the coloured portion of this card is divided into two equal parts, let us find what fraction of the whole square each of the two parts is.

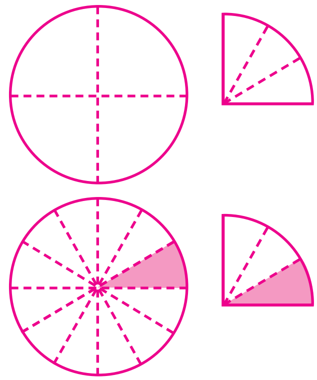


It is clear from the figure that each part is $\frac{1}{8}$ of the whole square.

This can also be written as $\frac{1}{4} \div 2$.

$$\therefore \frac{1}{4} \div 2 = \frac{1}{8}$$

Consider $\frac{1}{4}$th of the circle shown in the figure. If this is divided further into three equal portions, let us find what fraction of the whole circle each portion is.



It is clear that each portion is $\frac{1}{12}$ th of the whole circle.

$$\therefore \frac{1}{4} \div 3 = \frac{1}{12}$$

Now let us consider each of the above cases one by one.

$\frac{1}{2} \div 2 = \frac{1}{4}$. In addition, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. $\therefore \frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2}$

$\frac{1}{4} \div 2 = \frac{1}{8}$. In addition, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$. $\therefore \frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2}$

$\frac{1}{4} \div 3 = \frac{1}{12}$. In addition, $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$. $\therefore \frac{1}{4} \div 3 = \frac{1}{4} \times \frac{1}{3}$

Dividing a fraction by a number is the same as multiplying the fraction by the reciprocal of that number.

Example 1

Find the value of $\frac{1}{3} \div 2$.

$\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2}$ (multiplying by the reciprocal of 2)

$= \frac{1}{6}$

Example 2

Find the value of $\frac{4}{5} \div 3$.

$\frac{4}{5} \div 3 = \frac{4}{5} \times \frac{1}{3}$ (multiplying by the reciprocal of 3)

$= \frac{4}{15}$

Exercise 14.2

(1) Find the value of each of the following.

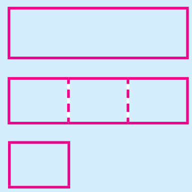
(i) $\frac{1}{5} \div 4$ (ii) $\frac{3}{4} \div 2$ (iii) $\frac{5}{7} \div 3$ (iv) $\frac{9}{10} \div 5$

• Dividing a whole number by a fraction

Let us now consider how a whole number is divided by a fraction. We can study this through examples.

Example 3

Find the value of $1 \div \frac{1}{3}$.



Let us consider the rectangular lamina shown here as one unit.

This unit has been divided into three equal parts. One of these parts is $\frac{1}{3}$.

Accordingly, there are three $\frac{1}{3}$ portions in one unit.

$\therefore 1 \div \frac{1}{3} = 3$

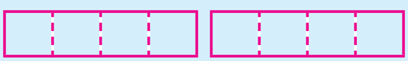
When 1 is multiplied by 3, which is the reciprocal of $\frac{1}{3}$, the same answer is obtained.

$$\therefore 1 \div \frac{1}{3} = 1 \times \frac{3}{1} = 3.$$

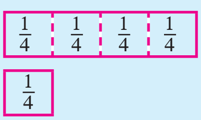
Example 4

Find the value of $2 \div \frac{1}{4}$.

Let us explain this by considering two rectangular shaped laminas of the same size. Let us consider each rectangular lamina as one unit.



When a lamina is divided into four equal parts, there are four $\frac{1}{4}$ in one unit.



Therefore, there are eight $\frac{1}{4}$ in two units.

Accordingly,

$$2 \div \frac{1}{4} = 8$$

$$2 \div \frac{1}{4} = 2 \times \frac{4}{1} = 8$$

Dividing a whole number by a fraction is the same as multiplying the number by the reciprocal of that fraction.

Example 5

Find the value of $3 \div \frac{1}{5}$.

$3 \div \frac{1}{5} = 3 \times 5$ (multiplying by the reciprocal)

$= 15$

Exercise 14.3

(1) Find the value of each of the following.

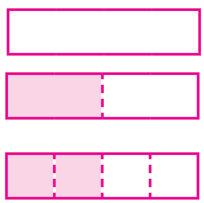
(i) $3 \div \frac{1}{4}$ (ii) $2 \div \frac{2}{5}$ (iii) $4 \div \frac{1}{2}$ (iv) $15 \div \frac{3}{5}$

14. 3 Dividing a fraction by a fraction

Consider $\frac{1}{2} \div \frac{1}{4}$.

Here we are trying to find out how many $\frac{1}{4}$ there are in $\frac{1}{2}$ a unit.

Let us illustrate this using a figure.



One unit

$\frac{1}{2}$ of the above unit

There are two $\frac{1}{4}$ in $\frac{1}{2}$ a unit.

Accordingly, $\frac{1}{2} \div \frac{1}{4} = 2$. To obtain this answer, $\frac{1}{2}$ should be multiplied by the reciprocal of $\frac{1}{4}$.

That is, $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$ (multiplying by the reciprocal of $\frac{1}{4}$)

$= \frac{4}{2} = 2$

Dividing a fraction by another fraction is the same as multiplying the first fraction by the reciprocal of the second fraction.

Example 1

Find the value of $\frac{1}{3} \div \frac{2}{5}$.

$\frac{1}{3} \div \frac{2}{5} = \frac{1}{3} \times \frac{5}{2}$ (multiplying by the reciprocal of $\frac{2}{5}$)

$= \frac{5}{6}$

Example 2

Find the value of $\frac{3}{7} \div \frac{6}{11}$.

$\frac{3}{7} \div \frac{6}{11} = \frac{\cancel{3}^{1}}{7} \times \frac{11}{\cancel{6}_{2}}$ (multiplying by the reciprocal of $\frac{6}{11}$)

$= \frac{11}{14}$

Exercise 14.4

(1) Find the value of each of the following.

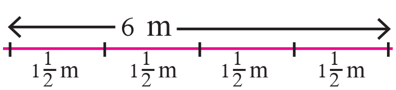
(i) $\frac{3}{8} \div \frac{3}{4}$ (ii) $\frac{15}{16} \div \frac{3}{4}$ (iii) $\frac{15}{28} \div \frac{3}{7}$ (iv) $\frac{10}{11} \div \frac{1}{11}$

(v) $\frac{6}{7} \div \frac{3}{7}$ (vi) $\frac{12}{7} \div \frac{3}{7}$ (vii) $\frac{4}{5} \div \frac{8}{9}$ (viii) $\frac{7}{8} \div \frac{7}{10}$

(ix) $\frac{3}{8} \div \frac{2}{5}$ (x) $\frac{2}{3} \div \frac{5}{7}$

14.4 Dividing a whole number by a mixed number

Let us find out how many pieces of wire of length $1\frac{1}{2}$ m can be cut from a wire of length 6 cm.



According to the figure, four pieces can be cut from the wire.
Accordingly, we can write $6 \div 1\frac{1}{2} = 4$.

Now let us simplify the expression $6 \div 1\frac{1}{2}$.

$6 \div 1\frac{1}{2} = 6 \div \frac{3}{2}$ (writing the mixed number $1\frac{1}{2}$ as an improper fraction)

$= \overset{2}{\cancel{6}} \times \frac{2}{\cancel{3}_{1}}$ (multiplying by the reciprocal of $\frac{3}{2}$)

$= 4$

- **Dividing a mixed number by a whole number**

Through the following example, let us establish how a mixed number is divided by a whole number.

Example 1

Find the value of $1\frac{1}{2} \div 6$.

$1\frac{1}{2} \div 6 = \frac{3}{2} \div 6$

$= \frac{\overset{1}{\cancel{3}}}{2} \times \frac{1}{\underset{2}{\cancel{6}}}$ (multiplying by the reciprocal of 6)

$= \frac{1}{4}$

14.5 Dividing a fraction by a mixed number

When dividing a fraction by a mixed number, the mixed number is first written as an improper fraction and then the fraction is multiplied by the reciprocal of this improper fraction.

Example 1

Find the value of $\frac{4}{5} \div 1\frac{1}{3}$.

$\frac{4}{5} \div 1\frac{1}{3} = \frac{4}{5} \div \frac{4}{3}$ (converting the mixed number into an improper fraction)

$= \frac{\overset{1}{\cancel{4}}}{5} \times \frac{3}{\underset{1}{\cancel{4}}}$ (multiplying by the reciprocal of $\frac{4}{3}$)

$= \frac{3}{5}$

- **Dividing a mixed number by a fraction**

Here, the mixed number is first written as an improper fraction. This improper fraction is then multiplied by the reciprocal of the fraction by which the mixed number is to be divided.

Example 2

Find the value of $1\frac{1}{3} \div \frac{4}{5}$.

$1\frac{1}{3} \div \frac{4}{5} = \frac{4}{3} \times \frac{5}{4}$

$= \frac{5}{3}$

$= 1\frac{2}{3}$

Exercise 14.5

(1) Find the value of each of the following.

(i) $3 \div 1\frac{1}{2}$ (ii) $7 \div 1\frac{1}{8}$ (iii) $15 \div 1\frac{1}{4}$ (iv) $18 \div 1\frac{2}{25}$

(v) $1\frac{1}{2} \div 3$ (vi) $1\frac{2}{5} \div 14$ (vii) $3\frac{2}{3} \div 22$ (viii) $5\frac{5}{6} \div 21$

(2) Find the value of each of the following.

(i) $\frac{3}{5} \div 2\frac{2}{5}$ (ii) $\frac{6}{7} \div 1\frac{1}{5}$ (iii) $\frac{8}{11} \div 3\frac{1}{5}$ (iv) $\frac{3}{8} \div 2\frac{1}{4}$

(v) $1\frac{4}{5} \div \frac{3}{5}$ (vi) $2\frac{1}{2} \div \frac{5}{7}$ (vii) $10\frac{2}{3} \div \frac{16}{27}$ (viii) $2\frac{3}{5} \div \frac{1}{2}$

(3) Hasim has packed 10 kg of sweetmeats into packets containing $1\frac{1}{4}$ kg each. Find the number of packets that he has made.

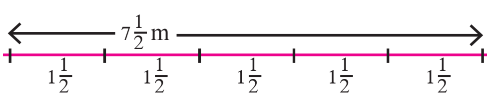
(4) A truck can transport $3\frac{1}{2}$ cubes of soil at a time. What is the minimum number of trips that needs to be made to transport 28 cubes of soil?

(5) Chalani needs to cut 21 m of fabric into $1\frac{3}{4}$ m pieces. How many such pieces can Chalani cut from this fabric?

(6) A volume of $31\frac{1}{2}$ *l* of paint in a barrel was poured equally into 7 containers. Find the amount of paint in each container.

14.6 Dividing a mixed number by a mixed number

Let us find out how many pieces of length $1\frac{1}{2}$ m can be cut from a rope of length $7\frac{1}{2}$ m.



It is clear from the figure that five pieces can be cut from the rope.

This can be written as $7\frac{1}{2} \div 1\frac{1}{2} = 5$.

Let us simplify $7\frac{1}{2} \div 1\frac{1}{2}$.

$$7\frac{1}{2} \div 1\frac{1}{2} = \frac{15}{2} \div \frac{3}{2} \quad \text{(converting the mixed number into an improper fraction)}$$

$$= \frac{\cancel{15}^{5}}{\cancel{2}_{1}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \quad \text{(multiplying by the reciprocal)}$$

$$= 5$$

When dividing a mixed number by a mixed number, the given mixed numbers are first converted into improper fractions, and the answer is obtained by the method of dividing a fraction by a fraction.

Example 1

Simplify $3\frac{1}{2} \div 1\frac{3}{4}$.

$$3\frac{1}{2} \div 1\frac{3}{4} = \frac{7}{2} \div \frac{7}{4}$$

$$= \frac{\cancel{7}^{1}}{{}_{1}\cancel{2}} \times \frac{\cancel{4}^{2}}{\cancel{7}_{1}} \quad \text{(multiplying by the reciprocal)}$$

$$= 2$$

Example 2

Simplify $2\frac{3}{5} \div 1\frac{7}{10}$.

$$2\frac{3}{5} \div 1\frac{7}{10} = \frac{13}{5} \div \frac{17}{10}$$

$$= \frac{13}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{17}$$

$$= \frac{26}{17}$$

$$= 1\frac{9}{17}$$

Exercise 14.6

(1) Simplify each of the following fractions.

(i) $2\frac{1}{4} \div 2\frac{2}{3}$ (ii) $7\frac{7}{8} \div 3\frac{1}{2}$ (iii) $6\frac{3}{5} \div 4\frac{5}{7}$

(iv) $7\frac{5}{8} \div 8\frac{5}{7}$ (v) $11\frac{1}{2} \div 2\frac{3}{4}$ (vi) $5\frac{1}{3} \div 2\frac{1}{2}$

(2) Fabric of length $2\frac{1}{4}$ m is required to sew a dress. What is the maximum number of such dresses that can be sewn from $56\frac{1}{4}$ m of fabric?

(3) The distance between two cities is $57\frac{1}{2}$ kilometers. A van spent $1\frac{9}{16}$ hours to travel from one city to the other. If it took the same amount of time to travel each kilometer, find how many kilometers it travelled in an hour?

(4) Among how many families can $148\frac{1}{2}$ kg of rice be distributed, so that each family gets $8\frac{1}{4}$ kg of rice?

Miscellaneous Exercise

(1) Simplify the following.

(i) $\frac{4}{5} \times 6$ (ii) $\frac{3}{7} \times 3$ (iii) $\frac{3}{8} \div 4$ (iv) $15 \div \frac{3}{10}$

(v) $8 \times \frac{3}{4}$ (vi) $5\frac{1}{4} \times 5$ (vii) $6\frac{3}{5} \div 3$ (viii) $8 \times 1\frac{1}{5}$

(ix) $7 \div 7\frac{1}{2}$ (x) $\frac{2}{3} \times \frac{7}{8}$ (xi) $\frac{3}{7} \times \frac{2}{3}$ (xii) $\frac{5}{9} \div \frac{7}{10}$

(xiii) $\frac{7}{8} \times \frac{4}{5} \times \frac{3}{7}$ (xiv) $\frac{2}{5} \times 1\frac{3}{7}$ (xv) $\frac{4}{9} \div 2\frac{1}{4}$ (xvi) $1\frac{3}{8} \div 1\frac{1}{7}$

(xvii) $1\frac{1}{2} \times 2\frac{2}{3}$ (xviii) $4\frac{2}{3} \div 1\frac{1}{7}$ (xix) $4\frac{1}{2} \times 3\frac{3}{5} \times 1\frac{1}{3}$ (xx) $3\frac{3}{4} \times 1\frac{2}{5} \times 1\frac{1}{7}$

Summary

- If the product of two numbers is 1, then each is the reciprocal of the other.
- Dividing a number by another number is the same as multiplying the first number by the reciprocal of the second number.